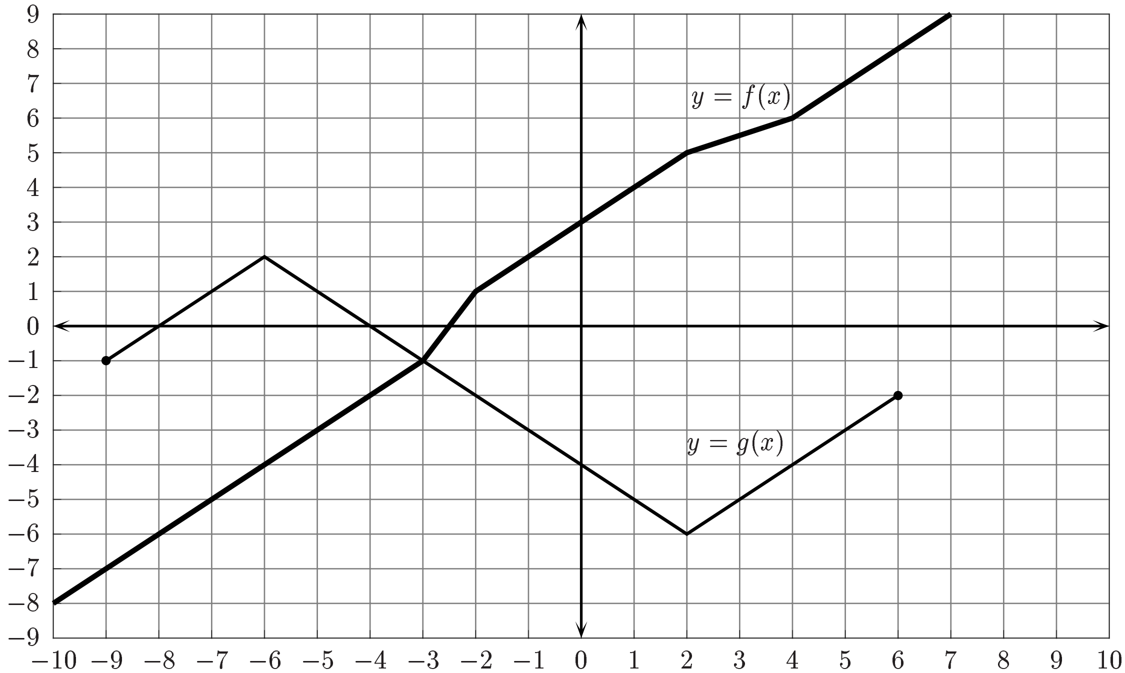
Name: ________________________ **PID:** _________________ **Section No:** _______

Page	Total	Score
2	24	
3	30	
4	24	
5	30	
Total	100	

1. Do not open this exam until you are instructed to do so.
2. *Without fully opening the exam,* check that you have pages 1 through 5 and that none are blank.
3. Fill in the information at the top of the page.
4. You will need a pen or pencil, **one** calculator and this booklet for the exam. Please clear everything else from your desk.
5. Calculators are not to be shared. Please do not ask your TA for any hints or any questions about the use of your calculator.
6. Please look to the board for possible corrections to this exam.
7. Do not spend too much time on a particular problem. Work the easier problems first.
8. The grading of this exam is based on your method. **Show all of your work.** (There are problems however that will be graded right or wrong.) If you need additional space, use the backs of the exam pages.
9. If a graphing utility is used to answer a question, please include a sketch of the graph (and the give the definition of the function you are graphing).
10. Place your answers in the boxes, where provided. Answers can be in any form unless specified otherwise. Calculator solutions must be accurate to within ± 0.0001 unless specified otherwise.
11. You will be given **exactly** 55 minutes for this exam.

1. (24 points) The graphs of $f(x)$ and $g(x)$ are shown in the sketch below. Use this sketch to answer the questions that follow.



(a) Evaluate $(fg)(5)$.

(b) Solve the equation $f(x) = g(x)$. $x =$

(c) Solve the inequality $g(x) > f(x)$.

(d) What is the **domain** of $g(x)$. (Express your answer using *interval* notation.)

(e) Evaluate $(f \circ g)(1)$.

(f) Evaluate $f^{-1}(8)$.

Problems 2 - 10 are multiple choice and are worth 6 points each. **Circle one answer for each question.**

2. If $f(x) = \dfrac{x}{2x-1}$ and $g(x) = \dfrac{1}{x} + 1$, find $(g \circ f)(x)$.

a) $\dfrac{x+1}{x+2}$ b) x c) $\left(\dfrac{x}{2x-1}\right)\left(\dfrac{1}{x}+1\right)$ d) $\dfrac{3x-1}{x}$ e) none of these

3. Compute the sum $\displaystyle\sum_{k=2}^{4} \frac{3k}{k+1}$.

a) $\dfrac{133}{20}$ b) $\dfrac{71}{20}$ c) $\dfrac{12}{5}$ d) 9 e) none of these

4. If $0 < x < 3$ then $\cos\left[\sin^{-1}\left(\frac{x}{3}\right)\right]$

a) $\dfrac{3-x}{3}$ b) $\dfrac{3}{\sqrt{9-x^2}}$ c) $-x$ d) $\dfrac{3}{x}$ e) $\dfrac{\sqrt{9-x^2}}{3}$

5. Solve the following inequality.

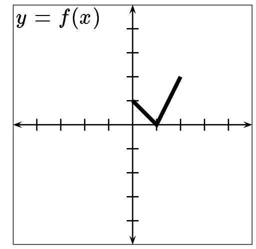
$$(x-1)(x+5)(3-x) > 0$$

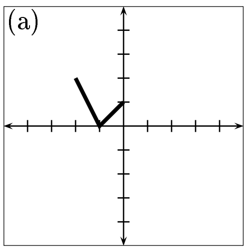
a) $-5, 1, 3$ b) $(-5, 1) \cup (3, \infty)$ c) $(-\infty, -5) \cup (1, 3)$ d) $(-\infty, 1) \cup (3, \infty)$ e) $x > 1, x > -5, x < 3$

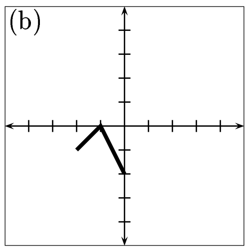
6. Find the vertex of $f(x) = x^2 + 8x + c$, where c is a real number.

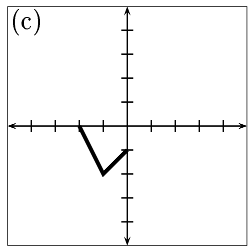
a) $(-4, c)$ b) $(-4, c-16)$ c) $(4, c+4)$ d) $(-4, c+16)$ e) none of these

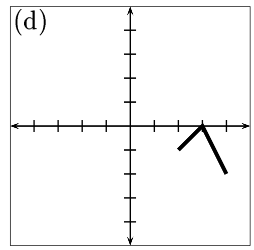
7. Let $y = f(x)$ be given by the sketch below. Circle the graph of $y = -f(x) - 2$. (*Note:* Each tick mark represents one unit.)

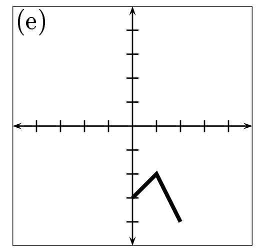












8. Find the average rate of change of $f(x) = \dfrac{250}{x}$ between $x = 2$ and $x = 4$.

a) 31.25 b) −31.25 c) 250 d) −0.032 e) −125

9. Find the amplitude, period, and phase shift of $g(x) = -2\cos(3x + \pi)$.

a) amplitude $= -2$, period $= 3$, phase shift $= \dfrac{\pi}{3}$

b) amplitude $= 2$, period $= 3$, phase shift $= \dfrac{-\pi}{3}$

c) amplitude $= 2$, period $= \dfrac{2\pi}{3}$, phase shift $= \dfrac{\pi}{3}$

d) amplitude $= 2$, period $= 3$, phase shift $= \dfrac{-\pi}{3}$

e) amplitude $= 2$, period $= \dfrac{2\pi}{3}$, phase shift $= \dfrac{-\pi}{3}$

10. Rewrite the expression below as the logarithm of a single quantity.

$$2\ln x - \frac{1}{2}(\ln y + 3\ln z)$$

a) $\dfrac{\ln x^2}{\ln\sqrt{y}\sqrt{z^3}}$ b) $\ln\dfrac{x^2\sqrt{z^3}}{\sqrt{y}}$ c) $\ln\left(x^2 - \sqrt{y} + \sqrt{z^3}\right)$ d) $\ln\dfrac{x^2 + \sqrt{z^3}}{\sqrt{y}}$ e) $\ln\dfrac{x^2}{\sqrt{yz^3}}$

11. (9 points) Simplify the expression $\sin\left(\frac{3\pi}{2} - x\right)$.

12. (9 points) Find the sum of the series below. Express your result as a rational number.

$$\sum_{k=1}^{\infty} 5\left(\frac{-2}{3}\right)^k$$

13. (12 points) Solve the system of equations below *algebraically*. Include a **complete** sketch of each graph in the box provided. YOUR GRAPH SHOULD INCLUDE ALL POINTS OF INTERSECTION.

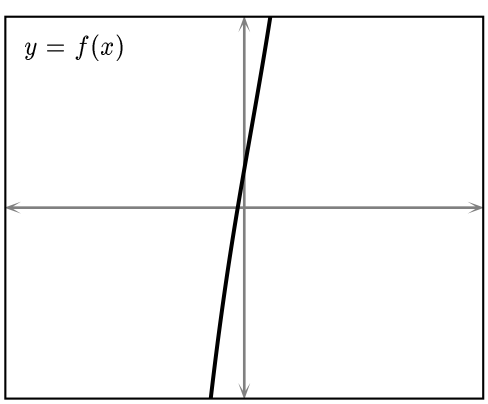
$$\begin{aligned} x^2 + y &= 4 \\ x + y &= 2 \end{aligned}$$

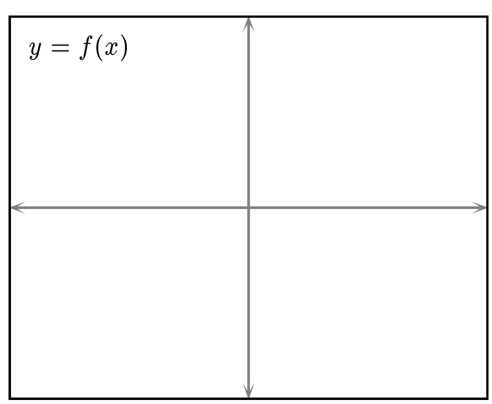
14. (10 points) Suppose that $\tan\alpha = \frac{5}{6}$ and α is an angle in quadrant III and $\sin\beta = \frac{8}{17}$ and β is an angle in quadrant II. Find the **exact** value of $\cos(\alpha - \beta)$.

15. (8 points) Use algebraic methods to find **exact** solutions to the equation below (i.e., solve the equation *without* using a graphing utility.).

$$2x + \sqrt{x} - 6 = 0$$

16. (8 points) Suppose that you used a graphing utility to sketch the graph of $f(x) = -3x^5 + 10x^4 + \ldots + 2$ and you got a picture as shown below on the left. In the box on the right, sketch what the **complete** graph might look like.





17. (8 points) The following statements are either *always* true (T) or *not always* true (F). Circle the one choice that best describes each.

    (a) If $x > y > 0$ then $\ln \frac{x}{y} = \ln(x - y)$. **T** **F**

    (b) If $i^2 = -1$ then $i^{-1} = -i$. **T** **F**

    (c) $5^{2x+1} = 5 \cdot 25^x$. **T** **F**

    (d) If $\frac{\pi}{2} < \theta < \pi$ then $\sin^{-1}(\sin \theta)) = \pi - \theta$. **T** **F**

18. (16 points) Fill in the blanks with the correct number or expression.

    (a) The graph of $g(x) = \frac{3x+2}{1-4x}$ has a vertical asymptote at ______________ and a horizontal asymptote at ______________.

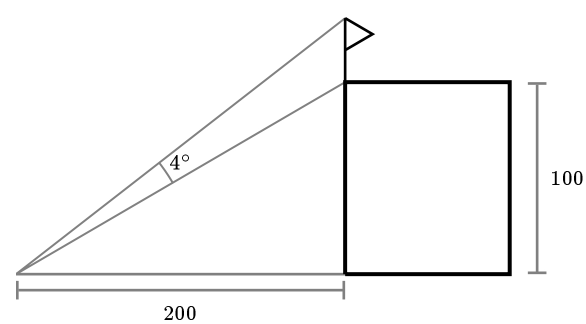
    (b) If $\cot \theta = \frac{-5}{2}$ and θ is in quadrant II then $\cos 2\theta =$ ______________.

    (c) If $g(x) = \sqrt{x+2}$ then $g^{-1}(x) =$ ______________ and the domain of $g^{-1}(x)$ is ______________.

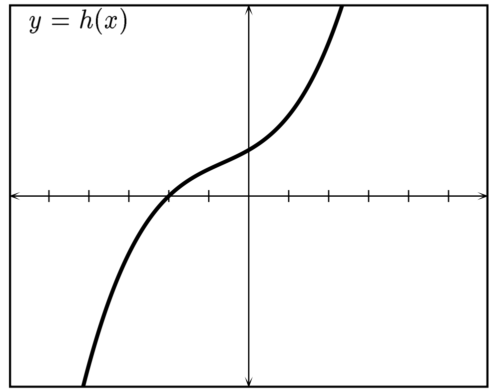
    (d) Let $p(x) = 100 + 300x^2 - 2x^3$. Then $p(x) \longrightarrow$ ______________ as $x \longrightarrow \infty$.

19. (10 points) A 10 gram sample of radioactive Berkelium, Bk-247, decays exponentially. If 9 grams remain after 210 years, what is the half-life of Bk-247 (i.e., how long does it take for the 10 gram sample to decay 5 grams?)? ANSWERS SHOULD BE ACCURATE TO WITHIN ± 0.1 YEARS.

20. (12 points) A flagpole sits on top of 100 tall building. Use the information in the sketch below to find the *length* of the flagpole. ANSWER TO THE NEAREST ± 0.1 FOOT.



21. (10 points) Let $h(x) = x^3 + 2x^2 + 6x + 12$. The graph of $y = h(x)$ is shown below. (*Note:* Each tick mark represents one unit.)



(a) Find all (real and complex) solutions to the equation $h(x) = 0$.

(b) Write $h(x)$ as a product of linear factors.

22. (10 points) Find all solutions to the equation below in the interval $[-2\pi, 2\pi]$.

$$\cos x + \sin 2x = 0$$

You may find the following formulas useful.

- $\sin 2x = 2\sin x\cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
- $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
- $Q(t) = Q_0\, e^{k\,t}$
- $P(t) = P_0\left(1+\frac{r}{n}\right)^{n\,t}$