Name: ____________________

Section: ____________ **Recitation Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Suppose $f(x) = \sqrt{x+7}$

   (a) (10 points) Use **the definition** of the derivative to find $f'(x)$.
   (Other methods will receive 0pts)

   Solution:

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\sqrt{x+h+7}-\sqrt{x+7}}{h}\\ &= \frac{(\sqrt{x+h+7}-\sqrt{x+7})}{h}\cdot\frac{(\sqrt{x+h+7}+\sqrt{x+7})}{(\sqrt{x+h+7}+\sqrt{x+7})}\\ &= \frac{x+h+7-(x+7)}{h}\cdot\frac{1}{(\sqrt{x+h+7}+\sqrt{x+7})}\\ &= \frac{h}{h}\cdot\frac{1}{(\sqrt{x+h+7}+\sqrt{x+7})}\\ &= \frac{1}{(\sqrt{x+h+7}+\sqrt{x+7})}\end{aligned}$$

   so therefore $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{(\sqrt{x+h+7}+\sqrt{x+7})} = \boxed{\frac{1}{2\sqrt{x+7}}}$

   (b) (2 points) What is the value of $f'(2)$?

   Solution: $f'(2) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

   (c) (2 points) Use your answer from (b) to find the equation of the tangent line to the curve $y = f(x)$ at the point $(2, f(2))$.

   Solution: $f(2) = \sqrt{9} = 3$ so the tangent line is given by

$$y - 3 = \frac{1}{6}(x-2) \quad \text{or} \quad y = \frac{1}{6}x + \frac{8}{3}$$

2. (14 points) Suppose that y and x satisfy the implicit equation

$$xy^3 + \frac{2x}{y} = 9.$$

Find the equation of the tangent line to the curve at the point $(3, 1)$.

Solution:

$$\begin{aligned}
(1)y^3 + x(3y^2 \cdot y') + \frac{2(y) - 2x(y')}{y^2} &= 0 && \text{(differentiate both sides)} \\
(1) + 3(3y') + 2 - 6y' &= 0 && \text{(plug in } x = 3,\ y = 1\text{)} \\
1 + 9y' + 2 - 6y' &= 0 \\
3y' &= -3 \\
y' &= -1
\end{aligned}$$

so the equation of the tangent line is given by

$$y - 1 = -1(x - 3) \quad \text{or} \quad y = -x + 4$$

3. (7 points) Find $f'(x)$ where $f(x) = (1+2x)^3 \cdot \sin(x)$

Solution:

$$f'(x) = 3(1+2x)^2(2) \cdot \sin(x) + (1+2x)^3 \cdot \cos(x)$$

4. (7 points) Find $g'(x)$ where $g(x) = \dfrac{\tan(3x)}{x^2}$

Solution:

$$g'(x) = \frac{\sec^2(3x)(3)(x^2) - \tan(3x)(2x)}{x^4}$$

or

$$g'(x) = \frac{3x\sec^2(3x) - 2\tan(3x)}{x^3}$$

or

$$g'(x) = 3\sec^2(3x) \cdot (x^{-2}) + \tan(3x) \cdot (-2x^{-3})$$

5. A particle moves according to the law of motion $s(t) = \dfrac{t^3}{3} + t^2 - 8t, \quad t \geq 0.$

   where t is measured in seconds and s in feet.

   (a) (5 points) For $t \geq 0$, when is the particle moving in the positive direction?

   Solution:

   $$v(t) = t^2 + 2t - 8 = (t+4)(t-2)$$

   so $v(t) = 0$ when $t = 2$. Breaking up a number line and choosing test points we see that

   $$v(1) = 1 + 2 - 8 = -5 < 0 \qquad\qquad v(3) = 9 + 6 - 8 = 7 > 0$$

   $v(t)$ $-\ -\ -\ -\ +\ +\ +\ +\ +\ +\ +$

   $[$0   1   2   3 $\rightarrow$

   so therefore the particle is moving in the positive direction $\boxed{\text{when } t \in (2, \infty)}$ (also accept $[2, \infty)$).

   (b) (3 points) What is the acceleration of the particle at $t = 5$ seconds?

   Solution: $a(t) = 2t + 2$. so $\boxed{a(5) = 12 \text{ ft/s}^2}$

   (c) (6 points) Find the total distance traveled by the particle from $t = 0$ to $t = 3$?

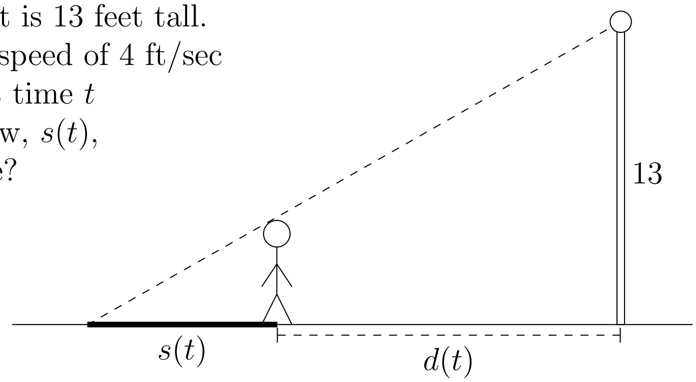
   Solution: Particle turns at $t = 2$ so we can calculate

   $$s(0) = 0 \qquad s(2) = -\frac{28}{3} \qquad s(3) = -6 = -\frac{18}{3}$$

   so

   $$|s(2) - s(0)| + |s(3) - s(2)| = \frac{28}{3} + \frac{10}{3} = \boxed{\frac{38}{3} \text{ ft}}$$

6. (14 points) A street light is at the top of a pole that is 13 feet tall. A woman 6 ft tall walks away from the pole with a speed of 4 ft/sec along a straight path. Her distance from the pole at time t is given by $d(t)$. How fast is the length of her shadow, $s(t)$, changing when she is 30 ft from the base of the pole?



Solution: Using similar triangles we get

$$\frac{s(t)+d(t)}{13} = \frac{s(t)}{6}$$

$$6s(t) + 6d(t) = 13s(t)$$

$$6d(t) = 7s(t)$$

$$6d'(t) = 7s'(t)$$

$$6(4) = 7s'(t)$$

$$\boxed{\frac{24}{7}\text{ ft/sec}} = s'(t)$$

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

7. (4 points) Consider the function $u(x) = f(x)g(x)$ where

$$f(-9) = 4 \qquad f'(-9) = 6 \qquad g(-9) = -5 \qquad g'(-9) = -4$$

Find $u'(-9)$

A. 2

B. -24

C. -46

D. -14

E. 1

8. (4 points) Where does $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 4x - 12}$ have a vertical asymptote?

A. $x = -1$

B. $x = 1$

C. $x = 6$

D. $x = -2$ and $x = 6$

E. $x = -2$

9. (4 points) Evaluate the limit: $\displaystyle\lim_{x \to 3} \frac{\frac{1}{x+1} - \frac{1}{4}}{x - 3}$

A. 4

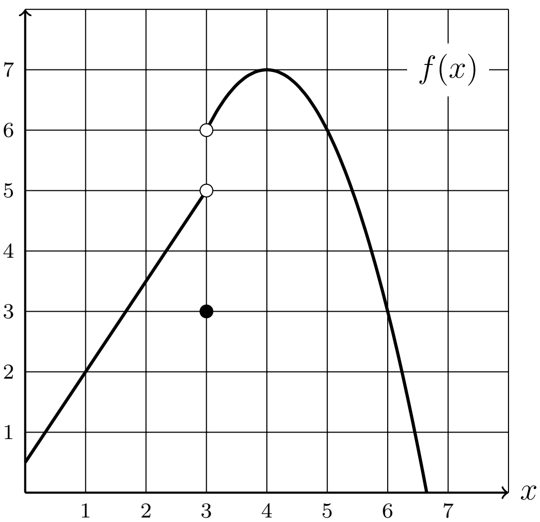
B. $1/4$

C. $-1/4$

D. $1/16$

E. $-1/16$

10. (4 points) The function of $f(x)$ is given by the graph below.



Evaluate the limit $\lim_{x\to 3} f(x)$

A. 5

B. 6

C. 3

D. 8

E. The limit does not exist.

11. (4 points) Indicate on what interval(s) $f(x) = \dfrac{x^2+9x+14}{x^2-7x-18}$ is continuous:

A. $(-\infty,-7)\cup(-7,\infty)$

B. $(-\infty,-2)\cup(-2,9)\cup(9,\infty)$

C. $(-\infty,-2)\cup(-2,\infty)$

D. $(-\infty,\infty)$

E. $(-\infty,9)\cup(9,\infty)$

12. (4 points) Find $f'(x)$ for $f(x) = \sin(3x^2+10x)$

A. $f'(x) = \cos(3x^2+10x)$

B. $f'(x) = \cos(6x+10)$

C. $f'(x) = \cos(3x^2+10x)\cdot(6x+10)$

D. $f'(x) = \cos(6x+10)\cdot(6)$

E. $f'(x) = \sin(6x+10)$

13. (4 points) On which of the following intervals must there be a solution to the equation $x^3 - 14 = 36 - 3x$?

A. $(1, 2)$

B. $(2, 3)$

C. $(3, 4)$

D. $(4, 5)$

E. $(5, 6)$

14. (4 points) Evaluate the limit $\lim_{x \to 0} \frac{\sqrt{x+36} - 6}{x}$

A. 36

B. $1/6$

C. $1/12$

D. 6

E. 1

15. (4 points) Evaluate the limit $\lim_{u \to 2} \frac{\sin(u^2 - 4)}{u - 2}$

A. 4

B. 2

C. 1

D. $1/2$

E. $1/4$

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	14	
7	12	
8	12	
9	12	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

- Distance from (x_1, y_1) to (x_2, y_2):

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.

Limits

- $\lim\limits_{x\to a} f(x) = L$ if for every $\varepsilon > 0$ there exists $\delta > 0$ so that $|f(x) - L| < \varepsilon$ when $|x-a| < \delta$.
- $\lim\limits_{x\to a} f(x)$ exists if and only if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

- $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(fg)' = f'g + fg'$
- $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$
- $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
- $(\sin x)' = \cos x$
- $(\cos x)' = -\sin x$
- $(\tan x)' = \sec^2 x$
- $(\sec x)' = \sec x \cdot \tan x$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE