Name: ____________________

Section: __________ **Recitation Instructor**: __________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. Calculate the following limits or show that they do not exist:

   (a) (4 points) $\lim_{x\to -1} \frac{x^2-1}{x+1} =$

   (b) (5 points) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{|x|}\right) =$

2. (5 points) Find the value of a that makes the function continuous at $x = 0$:

$$f(x) = \begin{cases} \dfrac{\sin(-8x)}{x} & \text{if } x < 0 \\ 3x + 6a - 7 & \text{if } x \geq 0 \end{cases}$$

3. (7 points) The length of a rectangle is decreasing at a rate of 4 cm/s and its width is increasing at a rate of 5 cm/s. When the length is 12 cm and the width 10 cm, how fast is the area of the rectangle changing? Is the area increasing or decreasing at that time? (*Include units.*)

4. Given $f(x) = x^2 + 10 \sin x$:

   (a) (1 point) Indicate the interval where the function is continuous.

   (b) (4 points) Prove that there is a number c such that $f(c) = 1$, using the Intermediate Value Theorem.

   (c) (2 points) Using (b) state an interval where c can be found.

5. Compute the derivatives of the following functions: (**DO NOT SIMPLIFY**)

(a) (4 points) $f(x) = x\sec(x)$

(b) (4 points) $g(x) = \dfrac{x^3+1}{6x^2+7}$

6. (6 points) Find the equation of the tangent line to the curve $y = \sin\left(\dfrac{\pi x^2}{4}\right)$ at the point $\left(1, \frac{\sqrt{2}}{2}\right)$.

7. (7 points) Given $y = \sqrt{x}$, use the limit definition of the derivative to compute y'.

8. (7 points) Consider $y^2 + xy + \dfrac{3}{y} = 4 + x^2$. Use implicit differentiation to find y'.

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

9. (4 points) Given $f(x) = |x - 5|$, which of the following statements is true?

   A. $f(x)$ is continuous and differentiable on $(-\infty, \infty)$.

   B. $f(x)$ is continuous on $(-\infty, \infty)$, and differentiable on $(-\infty, 5) \cup (5, \infty)$.

   C. $f(x)$ is continuous and differentiable on $(-\infty, 5) \cup (5, \infty)$.

   D. $f(x)$ is differentiable on $(-\infty, \infty)$, but not continuous at $x = 5$.

   E. $f(x)$ is not defined at $x = 5$.

10. (4 points) Suppose $f(x)$ is continuous and differentiable and that $f'(x) > 0$ always and $f(0) = 3$. What is true about $f(1)$?

    A. It is possible that $f(1) = 3$

    B. It must be that $f(1) < 3$

    C. It must be that $f(1) > 3$

    D. There is not enough information

11. (4 points) Given $x^2 + y^2 = 9$, which of the following is true?
    *(**Hint**: Use implicit differentiation or sketch the graph.)*

    A. $y' > 0$ always

    B. $y' < 0$ always

    C. $y' > 0$ in the I and III quadrants

    D. $y' > 0$ in the II and IV quadrants

    E. None of the above

12. Suppose the height of an object is modeled by $h(t) = 10t - 2t^2$ m, with time measured in seconds.

(a) (4 points) When does the object reach its maximum height?

A. 2.5 s
B. 25 s
C. 10 s
D. 4 s
E. None of the above.

(b) (4 points) What is the maximum height of the object?

A. 25 m
B. 12.5 m
C. 50 m
D. 2.5 m
E. None of the above.

(c) (4 points) What is the direction of the object at time $t = 4$ s?

A. downward
B. upward
C. There is not enough information.

13. (4 points) Use the squeeze theorem to evaluate: $\lim_{x\to 0}\sqrt{\frac{x^3+x^2}{\pi}}\sin\frac{\pi}{x}$

A. 1

B. 0

C. DNE

D. $\frac{1}{\pi}$

14. (4 points) Calculate the derivative of $f(x)=\cos(\tan x)$

A. $f'(x)=-\sin(\sec^2 x)$

B. $f'(x)=\sin(\tan x)\sec^2 x$

C. $f'(x)=-\sin(\tan x)\sec^2 x$

D. $f'(x)=\cos(\sec^2 x)$

E. $f'(x)=-\sin x\sec^2 x$

15. (4 points) The velocity of a particle moving back and forth along a straight line is given by $v(t)=2\sin(\pi t)+3\cos(\pi t)$, where time is measured in seconds.
What does $v'(t)=2\pi\cos(\pi t)-3\pi\sin(\pi t)$ represent?

A. The average rate of change of the position of the particle over any 1-second interval

B. The instantaneous rate of change of velocity

C. The speed at which the particle is moving

D. The average rate of change of the velocity of the particle over any 1-second interval

E. The instantaneous rate of change of position

More Challenging Question(s). Show all work to receive credit.

16. Newton's Law of Gravitation says that the magnitude of the force, F, exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) (4 points) Calculate $\frac{dF}{dr}$.

(b) (2 points) Explain the physical meaning of $\frac{dF}{dr}$.

(c) (8 points) Suppose the earth attracts an object with a force that decreases at the rate of 2 N/km when $r = 20,000$ km. How is this force changing when $r = 10,000$ km? (*Include units.*)

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1\end{aligned}$$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE