Name: ____________________

Section: __________ **Recitation Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. (6 points) Find the mot general antiderivative of the function $f(x) = x^5 - \sec(x)\tan(x) + \dfrac{1}{2\sqrt{x}}$

Solution: $F(x) = \dfrac{1}{6}x^6 - \sec(x) + \sqrt{x} + C$

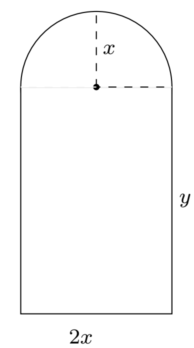
2. (8 points) Determine the value(s) of a such that:

$$\int_a^{a+1} (2x+3)\, dx = 10$$

Solution:

$$\begin{aligned}
\int_a^{a+1} (2x+3)\, dx &= x^2 + 3x\Big|_a^{a+1} \\
&= (a+1)^2 + 3(a+1) - a^2 - 3a \\
&= \cancel{a^2} + 1 + 2a + \cancel{3a} + 3 - \cancel{a^2} - \cancel{3a} \\
&= 2a + 4 = 10 \\
2a &= 6 \\
a &= 3
\end{aligned}$$

3. (14 points) A small region has the shape of a rectangle attached to a semicircle, so that the diameter of the semicircle is equal to the width of the rectangle. The perimeter is 2 m.
What is the width of such a region with the largest possible area?
What is the largest possible area?
Use one of the techniques of MTH 132 to justify that your solution indeed maximizes the area of the window.



Solution:

The function to be maximized is the area $A = 2xy + \frac{\pi}{2}x^2$

The perimeter is given by $P = 2x + 2y + \pi x = 2$, which solved for y yields:

$y = \frac{2 - 2x - \pi x}{2} = 1 - x - \frac{\pi}{2}x$

Use the expression above in the equation of the area:

$A = 2x\left(1 - x - \frac{\pi}{2}x\right) + \frac{\pi}{2}x^2 = 2x - 2x^2 - \frac{\pi}{2}x^2 = 2x - x^2\left(2 + \frac{\pi}{2}\right)$

Find the critical points by setting A'=0:

$A' = 2 - 2x\left(2 + \frac{\pi}{2}\right) = 0$

$x = \frac{1}{2 + \pi/2}$ or $x = \frac{2}{4 + \pi}$

$A = 2x - x^2\left(2 + \frac{\pi}{2}\right) = 2\frac{1}{2 + \pi/2} - \left(\frac{1}{2 + \pi/2}\right)^2$

Justification: Answers may vary.

One possible answer is: by the second derivative test $A'' = -2\left(\frac{1}{2 + \pi/2}\right) < 0$, meaning that the function A is concave down, therefore A is maximum at the critical point.

4. Given $f(x) = 5x^{2/3} - 2x^{5/3}$:

   (a) (4 points) Determine all its critical points

   Solution: $f'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3} = \frac{10}{3}x^{-1/3}(1-x)$

   $f' = 0$ or DNE for $x = 0, 1$

   (b) (3 points) Classify the critical points as local minima/maxima/neither

   Solution: $f'(x) > 0$ for $0 < x < 1$
   $f'(x) < 0$ for $x < 0 \cup x > 1$

   Therefore $f(x)$ has a local maximum at $x = 1$ and a local minimum at $x = 0$.

5. (7 points) Determine the absolute extrema for the function $f(x) = x - 2\sin x$ on the interval $[0, \pi]$. ($\sqrt{3} \simeq 1.73$.)

Solution:

By the EVT the absolute extrema can be found at the critical point(s), or at the endpoints of the given closed interval.

Find the critical point(s) by setting $f'(x) = 0$:

$f'(x) = 1 - 2\cos x = 0$; $\cos x = \frac{1}{2}$; $x = \frac{\pi}{3}$

Now compare:

$f(0) = 0$

$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\frac{\sqrt{3}}{2} = \frac{\pi}{3} - \sqrt{3} < 0$

$f(\pi) = \pi > 0$

Therefore $f(x)$ has its absolute maximum at $x = \pi$ and its absolute minimum at $x = \frac{\pi}{3}$

6. (6 points) Compute $\lim_{x\to\infty}(\sqrt{4x^2+3x}-2x)$

Solution:

$$\begin{aligned}\lim_{x\to\infty}(\sqrt{4x^2+3x}-2x) &= \lim_{x\to\infty}(\sqrt{4x^2+3x}-2x)\frac{\sqrt{4x^2+3x}+2x}{\sqrt{4x^2+3x}+2x}\\ &= \lim_{x\to\infty}\frac{\cancel{4x^2}+3x-\cancel{4x^2}}{\sqrt{4x^2+3x}+2x}\\ &= \lim_{x\to\infty}\frac{3x}{\sqrt{4x^2+3x}+2x}\\ &= \lim_{x\to\infty}\frac{3x}{\sqrt{4x^2+3x}+2x}\frac{1/x}{1/x}\\ &= \lim_{x\to\infty}\frac{3}{\sqrt{\frac{4x^2+3x}{x^2}}+\frac{2x}{x}}\\ &= \lim_{x\to\infty}\frac{3}{\sqrt{4+\dfrac{3}{x}}+2}=\frac{3}{4}\end{aligned}$$

7. (8 points) The acceleration of an object moving along the x-axis is $a(t)=3\sin t$. What are its velocity and position functions, $v(t)$ and $s(t)$, if $v(0)=1$ and $s(0)=3$?

Solution:

Find velocity:

$v(t)=-3\cos t+C$

$v(0)=-3+C=1$; $C=4$

$v(t)=-3\cos t+4$

Find position:

$s(t)=-3\sin t+4t+D$

$s(0)=D=3$

$s(t)=-3\sin t+4t+3$

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

8. (4 points) Using linear approximation, what is the best estimate of $\sqrt{4.1}$?

   A. $2 + \frac{1}{20}$

   B. $2 + \frac{1}{40}$

   C. $2 + \frac{1}{10}$

   D. 2

9. (4 points) Which function should you apply Newton's method to, in order to estimate $\sqrt{5}$?

   A. $f(x) = x^2 - 25$

   B. $f(x) = x - 5$

   C. $f(x) = x^2 - 5$

   D. $f(x) = \sqrt{5} - x^2$

10. (4 points) Determine all values of c satisfying the Mean Value Theorem for the function $f(x) = x^3 - 4x$ on the interval $-1 \leq x \leq 3$.

    A. $\pm\sqrt{\frac{7}{3}}$

    B. 3

    C. 5

    D. $\left(\frac{7}{3}\right)^{\frac{1}{2}}$

11. (4 points) If the length of the sides of a cube is measured to be 5 cm with a maximum error of 0.1 cm, use differentials to estimate the maximum error in the surface area.

A. 6 cm^2

B. 6 cm

C. 60 cm^2

D. 1.2 cm^2

E. 3 cm^2

12. (4 points) Select the true statements about the function $f(x) = \dfrac{x^3 + 4x}{(x+2)(x-1)}$:

A. The function has only two vertical asymptotes and no slant asymptotes

B. The function has only two vertical asymptotes and one slant asymptote

C. The function has no vertical asymptotes and only one slant asymptote

D. The function has only one vertical asymptote and one slant asymptote

13. (4 points) Compute $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n^2}$

A. 0

B. 2

C. DNE

D. 1

14. (4 points) The derivative of $f(x) = \int_1^{2x^2} \frac{\sin t}{1+t^2}\, dt$ is:

A. $\frac{4x \sin x}{1+x^2}$

B. $\frac{4x \sin(2x^2)}{1+4x^4}$

C. $\frac{\sin x}{1+x^2}$

D. $\frac{\sin(2x^2)}{1+4x^4}$

15. (4 points) Determine the value of $\int_{-5}^{0} |x+3|\, dx$. (*Hint: draw a picture of the region the integral represents, and find the area using simple formulas from geometry.*)

A. 5.5

B. -5.5

C. 6.5

D. -6.5

E. 0.5

16. (4 points) Estimate the net area A under the graph $y = x(x-2)$, between $x = 0$ and $x = 4$, using 4 rectangles of equal width, with heights of the rectangles determined by the height of the curve at left endpoints and at right endpoints.

A. $A = 2$ using left endpoints; $A = 10$ using right endpoints.

B. $A = 1$ using left endpoints; $A = 8$ using right endpoints.

C. $A = -1$ using left endpoints; $A = 9$ using right endpoints.

D. $A = 10$ using left endpoints; $A = 2$ using right endpoints.

More Challenging Question(s)

17. (14 points) The function $f(x)$ has *all* of the following properties:

1. $\lim_{x\to 2^-} f(x) = -\infty$
2. $\lim_{x\to 2^+} f(x) = \infty$
3. $f(2)$ DNE
4. $\lim_{x\to -\infty} f(x) = 0$
5. $f(-2) = 2$
6. $f(5) = 1$
7. $f(0) = 0$
8. $f'(x) > 0$ if $x < -2$ or $x > 5$
9. $f'(x) < 0$ if $-2 < x < 2$ or $2 < x < 5$
10. $f'(5) = 0$
11. $f'(-2) = 0$
12. $f''(x) > 0$ if $x < -3$ or $x > 2$
13. $f''(x) < 0$ if $-3 < x < 2$

Complete the following sentences:

(a) The domain of the function is: $(-\infty, 2) \cup (2, \infty)$.

(b) Such function must have vertical asymptote(s), with equation(s): x=2.

(c) There must be a horizontal asymptote with equation: $y = 0$.

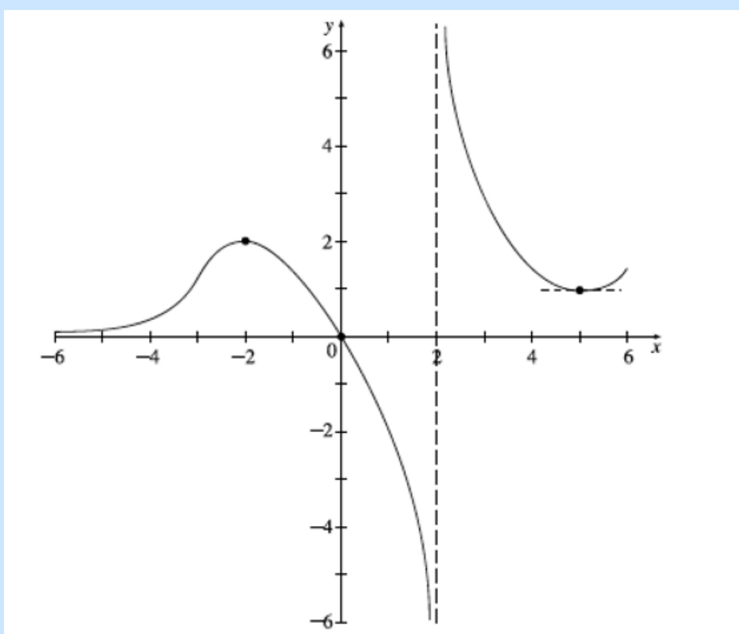
(d) There must be a local maximum of 2, and a local minimum of 1.

(e) Such function must have inflection point(s) at $x =$ -3.

(f) The function must be negative on $(0, 2)$.

(g) Sketch the curve.

Solution:



Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \int_a^x f(t)\,dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- $\sum_{i=1}^{n} c = cn$
- $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$