Name: ______________________________

Section: ______________ **(Recitation) Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions may have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. Compute the *derivatives* of the following functions: (**DO NOT SIMPLIFY**)

   (a) (7 points) $f(x) = x^2 \cos x - \sqrt[4]{x}$

   (b) (7 points) $g(x) = \dfrac{x}{\tan(3x-1)}$

2. (7 points) A snow ball melts so that its surface area decreases at a rate of 2 cm^2/min. How fast is its radius decreasing when the radius is 5 cm? (*Include units.*)

3. (7 points) Using the Intermediate Value Theorem prove that the following equation has a solution

$$\sqrt[3]{x} + x^2 = \cos \pi x.$$

4. Calculate the following limits or show that they do not exist:

   (a) (4 points) $\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2} =$

   (b) (4 points) $\lim_{x \to 2^-} \frac{x\left(x^2-4\right)}{\left|x^2-4\right|} =$

5. (6 points) Consider the function $f(x) = \frac{1}{2x+1}$. Use the limit definition of the derivative to show that $f'(x) = \frac{-2}{(2x+1)^2}$. (Your calculation must include computing a limit.)

6. Suppose you are on the moon and you shoot an arrow straight upward. Its height in meters after t seconds is $s(t) = -3t^2 + 2t + 1$.

   (a) (2 points) Find the velocity at time t.

   (b) (4 points) With what velocity (include units) will the arrow hit the ground?

7. (8 points) Find the equation of the tangent line to the curve defined by the equation $\boxed{x \sin(x + y) = y - \pi}$ at the point (π, π).

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

8. (4 points) Given a continuous function f on $(-\infty, \infty)$ with $f(0) = 6$, $f(2) = 1$, and $f(9) = -2$. Which of the following statements is necessarily true?

   A. There is c in [0,2] with $f(c) = 0$.

   B. There is c in [-1,5] with $f(c) = 0$.

   C. There is c in [0, 2] with $f(c) = 3$.

   D. There are a and b in $(-\infty, \infty)$ with $a \neq b$ and f(a)=f(b).

   E. None of the above is necessarily true.

9. (4 points) Suppose $f(x)$ is continuous and differentiable and that $f'(x) < 0$ for all x, and $f(1) = 4$. What is true about $f(0)$?

   A. It is possible that $f(0) = 4$.

   B. It must be that $f(0) < 4$.

   C. It must be that $f(0) > 4$.

   D. There is not enough information.

10. (4 points) The domain of $f(x) = \sqrt{\dfrac{x}{x+2}}$ is:

    A. $(-\infty, -2) \cup (-2, 0]$

    B. $(-\infty, -2) \cup [0, \infty)$

    C. $(-\infty, -2)$

    D. $(-2, 0]$

    E. $(-2, \infty)$

11. (4 points) Suppose

$$f(2) = -3,\ g(2) = 5,\ f'(2) = 2,\ g'(2) = 6.$$

Then the derivative of $\dfrac{2g(x)}{1+f(x)}$ at $x = 2$ is

A. -11

B. 6

C. 4

D. 44/9

E. 22

12. (4 points) Calculate the derivative of $f(x) = \tan(\sin(x^2))$,

A. $f'(x) = -\sec^2(\sin(x^2))\cos(x^2)\,(2x)$

B. $f'(x) = \sec^2(\cos(2x))$

C. $f'(x) = (\sec^2 x)\cos(x^2)\,(2x)$

D. $f'(x) = \sec^2(\sin(x^2))\cos(x^2)\,(2x)$

E. $f'(x) = -(\sec^2 x)\cos(x^2)\,(2x)$

13. (4 points) For what value of c the function $f(x) = \begin{cases} x^2 - 10, & x \le c \\ 10x - 35, & x > c \end{cases}$ is continuous everywhere.

A. $c = 10$

B. $c = 5$

C. $c = \sqrt{10}$

D. $c = 20$

E. None of the above

14. (4 points) Evaluate: $\lim_{x \to 0} \dfrac{\sin(x^2 + 7x)}{2\,x}$

A. 0

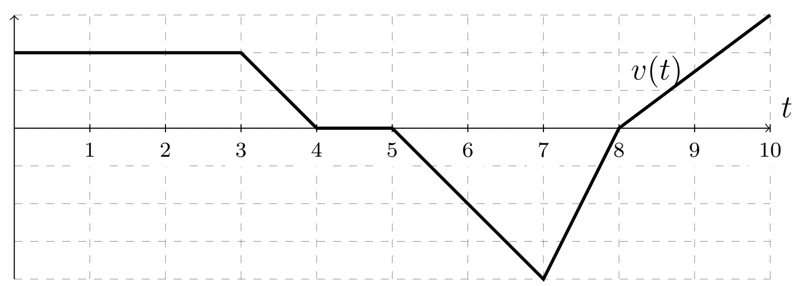
B. $\frac{7}{2}$

C. 1

D. 7

E. Does not exist

The following graph shows **the velocity** of a particle moving in a straight line for $t \in [0, 10]$.
Use it to answer the following two questions



15. (4 points) For what values of t is the particle moving forward?

A. (7, 10)

B. $(0, 3) \cup (4, 5)$

C. $(5, 8)$

D. $(3, 4) \cup (5, 7)$

E. $(0, 4) \cup (8, 10)$

16. (4 points) Which of the following statements is true of the particle on the time interval $(5, 7)$?

A. It is moving forwards and slowing down.

B. It is moving forwards and speeding up.

C. It is moving backwards and slowing down.

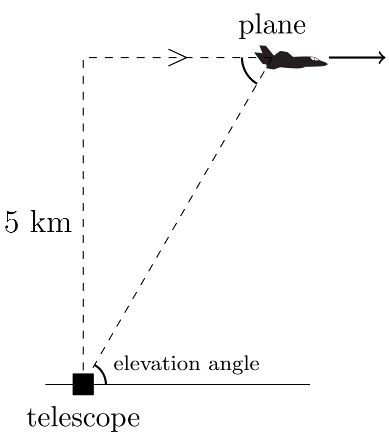
D. It is moving backwards and speeding up.

E. None of the above.

More Challenging Question(s).

17. (4 points) Calculate the following limit: $\lim_{x \to 0^+} \left[x \sin \frac{1}{x} \right] =$

18. (10 points) A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\frac{\pi}{3}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ rad/min. How fast is the plane traveling at that time?



Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

- Distance from (x_1, y_1) to (x_2, y_2):

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.

Limits

- $\lim\limits_{x\to a} f(x)$ exists if and only if $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$
- $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE