Name: ______________________________

Section: ______________ **Recitation Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 9.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Calculate the first derivative of the following functions.

(a) (4 points) $f(x) = \sqrt[3]{x} + \dfrac{1}{x^2}$

Solution:

$$f(x) = x^{1/3} + x^{-2}$$
$$f'(x) = \frac{1}{3}x^{-2/3} - 2x^{-3}$$

(b) (4 points) $g(x) = \dfrac{3x^4}{\tan x}$

Solution:

$$g'(x) = \frac{(12x^3)(\tan x) - (3x^4)(\sec^2 x)}{\tan^2 x}$$

(c) (6 points) $h(x) = \cos^3 x$

Solution:

$$h(x) = (\cos x)^3$$
$$h'(x) = 3(\cos x)^2(-\sin x)$$

2. Let $f(x) = -x(x-4) = -x^2 + 4x$ to answer the following questions:

(a) (4 points) Calculate the average rate of change of f over the interval [1, 2].

Solution:

$$\frac{f(2) - f(1)}{2-1} = \frac{(-4+8) - (-1+4)}{1} = 4 - 3 = 1$$

(b) (6 points) Calculate $f'(1)$ using the **definition** of the derivative. *(other methods will receive 0pts)*

Solution:

$$\begin{aligned} f'(1) &= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h\to 0} \frac{-(1+h)^2 + 4(1+h) - 2}{h} \\ &= \lim_{h\to 0} \frac{-1 - 2h - h^2 + 4 + 4h - 3}{h} \\ &= \lim_{h\to 0} \frac{-2h - h^2 + 4h}{h} \\ &= \lim_{h\to 0} -2 - h + 4 = 2 \end{aligned}$$

(c) (2 points) Write an equation of the tangent line through the point $(1, f(1))$.

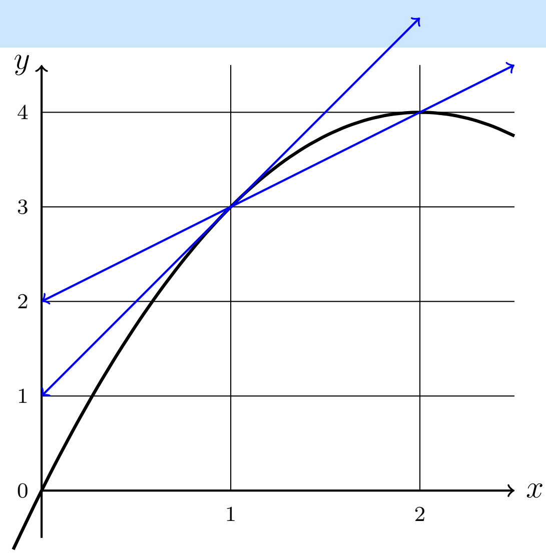
Solution:

$$y - 3 = 2(x-1)$$

(d) (2 points) Use the graph on the right to sketch:

- a secant line through $(1, f(1))$ and $(2, f(2))$
- a tangent line through the point $(1, f(1))$

Results from parts (a)-(c) might be helpful.

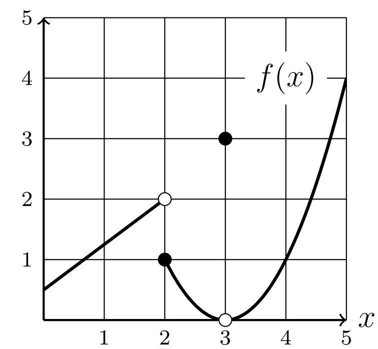


3. Use the graph of $f(x)$ shown below to answer the following questions.

(a) (4 points) Evaluate the limits:

$$\lim_{x\to 2^+} f(x) = 1 \qquad \lim_{x\to 2} f(x) = DNE$$

$$\lim_{x\to 3} f(x) = 0 \qquad f(3) = 3$$



(b) (4 points) Is $f(x)$ is continuous at $x = 3$? Use the definition of continuity to explain your answer.

Solution: No it is not

$$3 \neq 0$$
$$\lim_{x\to 3} f(x) \neq f(3)$$

4. The height of a projectile (in feet) is given by the function $h(t) = -16t^2 + 64t + 5$.

(a) (2 points) Is the projectile moving up or down at $t = 1$? *Show your work!*

Solution:

$$v(t) = -32t + 64$$
$$v(1) = -32(1) + 64 = 32 \text{ ft/sec}$$

Since the velocity is positive the particle is moving up.

(b) (4 points) What is the maximum height of the projectile? *Include units!*

Solution:

$$v(t) = -32t + 64$$
$$0 = -32t + 64$$
$$t = 2$$

$$h(2) = -16(4) + 64(2) + 5 = -64 + 128 + 5 = 69 \text{ feet}$$

5. (7 points) Suppose that y and x satisfy the implicit equation

$$2x + 2y + x^2y^3 = 2.$$

Find the the derivative $\frac{dy}{dx}$ at the point $(-2, 1)$.

Solution:

$$\begin{aligned}
2 + 2y' + (2x)y^3 + x^2(3y^2y') &= 0 && \text{(By taking derivative of both sides.)}\\
2 + 2y' + 2(-2)(1)^3 + (-2)^2(3(1)^2y') &= 0 && \text{(By plugging in } x = -2 \text{ and } y = 1\text{)}\\
2 + 2y' - 4 + 12y' &= 0\\
14y' &= 2\\
y' &= 1/7
\end{aligned}$$

6. (7 points) You are blowing air into a spherical balloon at a constant rate of 11 in^3/sec. How fast is the radius of the balloon growing when the balloon has a radius of 4 inches?

Solution:

$$\begin{aligned}
V(t) &= \tfrac{4}{3}\pi r(t)^3\\
V'(t) &= 4\pi r(t)^2 \cdot r'(t)\\
11 &= 4\pi 4^2 \cdot r'(t)\\
\frac{11}{64\pi}\ \text{in/sec} &= r'(t)
\end{aligned}$$

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

7. (4 points) If $f(x)$ is a differentiable function, which of the following statements about $f'(1)$ is true?

   (I) $f'(1)$ is the y-value at $x = 1$.

   (II) $f'(1)$ is the average rate of change at $x = 1$.

   (III) $f'(1)$ is the instantaneous rate of change at $x = 1$.

   (IV) $f'(1)$ is the slope of the secant line at $x = 1$.

   (V) $f'(1)$ is the slope of the tangent line at $x = 1$.

   A. (I) only B. (II) only C. (III) only D. (II) and (IV) E. (III) and (V)

8. (4 points) Which equation is a tangent line at $x = 2$ if $f(2) = 4$ and $f'(2) = -3$?

   A. $y - 2 = -3(x - 4)$

   B. $y + 3 = 4(x - 2)$

   C. $y - 4 = -3(x - 2)$

   D. $y - 2 = 4(x + 3)$

   E. $y - 4 = 2(x + 3)$

9. (4 points) Evaluate the limit: $\lim_{x \to 2^-} \dfrac{3x(x-2)}{|x-2|}$

   A. The limit does not exist (DNE).

   B. -1

   C. 1

   D. 6

   E. -6

10. (4 points) The functions $f(x)$ and $g(x)$ are given by the graph below.

Suppose $h(x) = f(x) \cdot g(x)$. Calculate $h'(2)$:

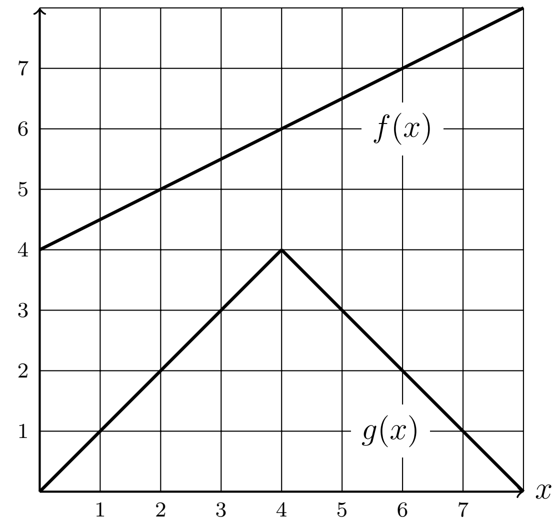
A. $\frac{1}{2}$

B. 6

C. 2

D. 8

E. The limit does not exist.



11. (4 points) Calculate: $\lim_{x \to -2} \dfrac{x^2 + 5x + 6}{x^2 + x - 2}$

A. $-\frac{1}{4}$

B. $\frac{0}{0}$

C. $-\frac{1}{3}$

D. ∞

E. $-\infty$

12. (4 points) Which of the following equations are vertical asymptotes of $g(x) = \dfrac{x^2 - 2x - 3}{x^2 - 4x + 3}$?

A. $x = 1$

B. $x = 3$

C. $x = 1$ and $x = 3$

D. $x = -3$

E. None of the above

13. (4 points) Calculate the derivative of $f(x) = x^2 \cos x$

A. $f'(x) = -2x \sin x$

B. $f'(x) = 2x \sin x$

C. $f'(x) = 2x \cos x - x^2 \sin x$

D. $f'(x) = 2x \cos x + x^2 \sin x$

E. $f'(x) = 2x \cos x$

14. (4 points) On which of the following intervals must there be a solution to the equation $\sqrt{x} + 1 = x^2$?

A. $(0, 1)$

B. $(1, 2)$

C. $(2, 3)$

D. $(3, 4)$

E. $(4, 5)$

15. (4 points) Evaluate the limit $\lim_{x \to 64} \frac{\sin(\sqrt{x} - 8)}{x - 64}$

A. 1/64

B. 16

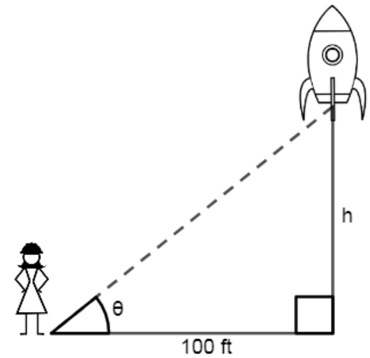
C. 64

D. 1/16

E. 0/0

More Challenging Question(s). Show all work to receive credit.

16. The goal of this problem is to determine the velocity of a rocket using a measurement that can be taken by an observer on the ground. Suppose the observer is 100ft from the launch site of a rocket that is launched straight in the air, and the observer is able to measure the angle of elevation θ of the rocket. An illustration of the situation and a table of the measurements is given to the right. Using the table of data answer the following questions.



Time (t sec)	**Angle of Elevation (θ radians)**
0	0
0.1	0.29
0.2	0.81
0.3	0.95
0.4	$1.05 \approx \frac{\pi}{3}$
0.5	1.15
0.6	1.22

(a) (7 points) Estimate the rate of change $\frac{d\theta}{dt}$ at $t = 0.4$ sec. *Include units!*

Solution: The best estimate for the instantaneous rate of change we can get with a table of data is the average rate of change over a small interval. In this case

$$\frac{1.05 - .95}{.4 - .3} = \frac{.1}{.1} = 1$$
$$\frac{1.15 - 1.05}{.5 - .4} = \frac{.1}{.1} = 1$$

so we estimate that $\dfrac{d\theta}{dt} = 1$ radians/sec when $t = 0.4$ sec.

(b) (7 points) Estimate the vertical velocity of the rocket at $t = 0.4$ sec. *Include units!*

Solution: Given the picture to the right we can setup the equation

$$\tan(\theta(t)) = \frac{h(t)}{100} \qquad \text{(differentiating both sides we get)}$$
$$\sec^2(\theta(t)) \cdot \theta'(t) = \frac{h'(t)}{100} \qquad \text{(now we plug in our specific values)}$$
$$\sec^2(\tfrac{\pi}{3}) \cdot (1) = \frac{h'(t)}{100} \qquad \text{(and solve)}$$
$$400 \text{ ft/sec } = h'(t)$$