Name: ____________________

Section: ____________ **Recitation Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. (7 points) The region in the first quadrant bounded by the $y = x^3$, $y = \sqrt{x}$, is rotated about the $y-$axis. Compute the volume of the resulting solid of revolution.

2. (7 points) Find the function $y = y(x)$ which solves the initial value problem

$$\frac{dy}{dx} = \frac{y}{2 + x^2} \quad \text{and} \quad y(0) = 1.$$

3. (7 points) A swimming pool built in the shape of a rectangular prism 3 m deep, 5 m wide and 10 m long is filled 1 m below the top. How much work is required to pump all the water into a drain at the top edge of the pool ? (water weighs 9800 N/m^3).

4. (7 points) Two positively charged particles repel each other with a force of $F(x) = \dfrac{1}{x^2}$ where x is the distance between the two. One particle is held fixed while the other is pushed from a point 1 m away from the first particle to a point 0.1 m away from the first particle. How much work is done ?

5. (7 points) Find the derivative of $f(x) = (x+1)^{\ln(\mathrm{x})}$.

6. (7 points) Evaluate $\displaystyle\int \frac{3x^2+2}{x^3+2x-5}\,dx$.

7. (8 points) After an accident at a nuclear facility a sample of sea water was tested for radioactive contamination, and it showed an elevated level of 27 Bq/m^3 (Becquerel per cubic meter, a measure of radiation) primarily due to Cesium-137 which has a half life of 30 years. How many years does it take for the radiation to return to its natural level of 1 Bq/m^3 ? (Use the approximate values 10/3 and 7/10 for $\ln 27$ and $\ln 2$ respectively)

8. (6 points) Evaluate $\lim_{x \to 0} \frac{\ln(\sec x)}{x^2}$. (*If you use l'Hospital's Rule, explicitly state your reasoning.*)

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

9. (4 points) What is the value of x if $\displaystyle\int_1^{x^2} \frac{dt}{t} = -1$?

    A. e

    B. $\dfrac{1}{e}$

    C. $\dfrac{1}{\sqrt{e}}$

    D. $\sqrt{e}$

    E. 0

10. (4 points) What is the value of $\cos(\tan^{-1}(3))$?

    A. $\dfrac{1}{\sqrt{10}}$

    B. $\dfrac{3}{\sqrt{10}}$

    C. $\dfrac{1}{\sqrt{3}}$

    D. $\dfrac{10}{\sqrt{3}}$

    E. $\dfrac{2}{\sqrt{10}}$

11. (4 points) What is the derivative of $y = \sin^{-1}(\ln x)$?

    A. $\dfrac{1}{x\sqrt{1-x^2}}$

    B. $\dfrac{1}{x\sqrt{1-\ln^2 x}}$

    C. $\dfrac{1}{\ln x\sqrt{1-x^2}}$

    D. $\dfrac{1}{x(1+x\ln^2 x)}$

    E. $\dfrac{1}{\ln x(1+x^2)}$

12. (4 points) Evaluate $\int_0^{\ln 2} \sinh 2x \; dx$.

A. $\frac{17}{16}$

B. $\frac{1}{2}$

C. $\frac{15}{16}$

D. $\frac{9}{16}$

E. $\frac{1}{16}$

13. (4 points) Evaluate $\int \sin^4 x \cos^3 x \; dx$.

A. $\frac{1}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

B. $\frac{1}{7}\sin^7 x - \frac{1}{5}\sin^5 x + C$

C. 0

D. $-\frac{1}{20}\cos^5 x \sin^4 x + C$

E. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$

14. (4 points) Let $f(x) = x^3 + 2\sqrt{x} + 2$. What is $(f^{-1})'(5)$?

A. $7/2$

B. $75 + \frac{1}{\sqrt{5}}$

C. $1/5$

D. $1/4$

E. $1/2$

15. (4 points) Determine the values for A and B, such that $y = \dfrac{1}{x(10+x)} = \dfrac{A}{x} + \dfrac{B}{10+x}$.

A. $A = \frac{1}{10}, B = 10$

B. $A = 10, B = \frac{1}{10}$

C. $A = 1, B = 1$

D. $A = \frac{1}{10}, B = \frac{1}{10}$

E. $A = \frac{1}{10}, B = -\frac{1}{10}$

16. (4 points) Which of the following integrals is improper?

A. $\displaystyle\int_0^1 \frac{dt}{\ln(2+x)}$

B. $\displaystyle\int_0^1 \frac{1}{x^2-4}\ dx$

C. $\displaystyle\int_0^1 \frac{1}{t} dx$

D. $\displaystyle\int_0^1 \frac{1}{x^2-1} dx$

E. $\displaystyle\int_{-1}^1 \frac{1}{x^2-4} dx$

17. (4 points) Evaluate $\displaystyle\int_0^{\pi/2} x \sin x\ dx$.

A. 1

B. $\frac{\pi}{2}$

C. -1

D. $-\frac{\pi}{2}$

E. 0

More Challenging Questions

18. (4 points) **Multiple Choice** Consider the following two unrelated statements:

  1. If $f(t) \leq g(t)$ and $\displaystyle\int_1^\infty f(t)\, dt$ diverges, then $\displaystyle\int_1^\infty g(t)\, dt$ diverges.
  2. $\left(\dfrac{\log_3 x}{\log_7 x}\right)' = 0$

  A. both statements are true
  B. 1. is true and 2. is false
  C. 1. is false and 2. is true
  D. both statements are false

19. (10 points) Compute $\displaystyle\int \sqrt{x}\tan^{-1}\sqrt{x}\,dx$. Show all your work!

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$
- $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\displaystyle\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad$ for $a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

Derivatives

- $\dfrac{d}{dx}(\sinh x) = \cosh x \qquad \dfrac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$