Name: ____________________

Section: __________ **Recitation Instructor**: __________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- You may detach the formula sheet(s) found at the end of this exam.

ACADEMIC HONESTY

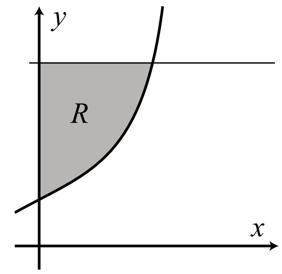
- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

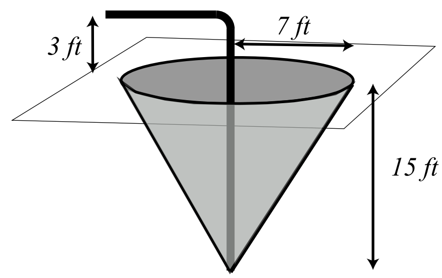
Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. (7 points) Let R be the region in the first quadrant below the line $y = 3$ and above the curve $y = e^x$, (shown in the picture). Find the volume of the solid formed by revolving R about the x-axis.



2. (7 points) An underground tank is a cone with radius 7 feet and height 15 feet. The tank is full of water (water weighs 62.5 lb/ft^3). How much work is required to empty the tank by pumping the water to a height of 3 ft above ground level?

   Write your answer as an integral, but **do not evaluate the integral.**



3. (7 points) Differentiate the function $f(x) = \left(\ln x\right)^{\cosh(x)}$.

4. (7 points) Use partial fractions to find $\displaystyle\int \frac{x+7}{x^2-x-2}\,dx$.

5. (6 points) Evaluate the definite integral $\displaystyle\int_e^{e^2} \frac{1}{x\sqrt{\ln x}}\,dx$.

6. (a) (2 points) Find the derivative of $y = x\ln(x) - x$.

(b) (6 points) Integrate $\displaystyle\int (\ln x)^2\,dx$.

7. (7 points) For the function $y = \sin^{-1}(x)$, derive the formula $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ using either implicit differentiation or the general formula for $(f^{-1})'$. *Show all work and make your reasoning clear.*

8. (7 points) Integrate $\displaystyle\int \frac{1}{\left(9+x^2\right)^{3/2}}\,dx$.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

9. (4 points) The derivative of $y = \cosh(x^2) + \sinh(2x)$ is:

   A. $2x\,\sinh(x^2) + 2\cosh(2x)$

   B. $-2x\,\sinh(x^2) + 2\cosh(2x)$

   C. $2x\,\sinh(2x) - 2\cosh(x^2)$

   D. $2x\,\sinh(x^2) - 2\cosh(2x)$

   E. $2x\,\sinh(2x) + 2\cosh(x^2)$

10. (4 points) The function $f(x) = \displaystyle\int_1^{e^x} \frac{1}{t}\,dt$ is equal to

    A. $e^x - 1$ B. $\dfrac{1}{e^x} - 1$ C. $1 - \dfrac{1}{e^{2x}}$ D. x E. $x - 1$

11. (4 points) What is the solution $y(x)$ of the initial value problem $y' = \dfrac{x^2+1}{2y}$ with $y(0) = 1$?

    A. $y = e^{\frac{1}{6}x^3 + \frac{1}{2}x}$

    B. $y = \sqrt{\frac{1}{3}x^3 + x + 1}$

    C. $y = \frac{1}{6}x^3 + \frac{1}{2}x + 1$

    D. $y = 1 - \sqrt{x^3 + x}$

    E. $y = \frac{1}{2}x^2 + 1$

12. (4 points) Evaluate $\displaystyle\int_0^{\pi/4} \tan^6(x)\sec^4(x)\,dx$.

A. $\dfrac{1}{9}+\dfrac{1}{7}$ B. $\dfrac{1}{8}+\dfrac{1}{6}$ C. $\dfrac{1}{7}-\dfrac{1}{5}$ D. $\dfrac{1}{7}+\dfrac{1}{5}$ E. 0

13. (4 points) What is the form of the partial fraction decomposition of $\dfrac{1}{(x^2+1)(x-3)^2}$?

A. $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-3}$

B. $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{(x-3)^2}$

C. $\dfrac{A}{x^2+1}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$

D. $\dfrac{A}{x+1}+\dfrac{B}{x^2+1}+\dfrac{C}{x-3}+\dfrac{D}{(x-3)^2}$

E. $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-3}+\dfrac{D}{(x-3)^2}$

14. (4 points) Compute the integral $\displaystyle\int \frac{\sin x}{3+\cos x}\,dx$.

A. $\ln(3+\cos x)+C$

B. $-\ln(3+\cos x)+C$

C. $\ln(3+\sec x)+C$

D. $\frac{1}{3}x+\ln(|\sec(x)|)+C$

E. $-\frac{1}{3}\cos(x)+\ln(|\sec(x)|)+C$

15. (4 points) Is the improper integral $\displaystyle\int_1^\infty \frac{2+\sin(x)}{x^2}\,dx$ convergent?

A. It is convergent because $0 \le \dfrac{1}{x^2} \le \dfrac{2+\sin(x)}{x^2}$ and $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ is convergent.

B. It is convergent because $0 \le \dfrac{2+\sin(x)}{x^2} \le \dfrac{3}{x^2}$ and $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$ is convergent.

C. It is divergent because $0 \le \dfrac{2+\sin(x)}{x^2} \le \dfrac{3}{x^2}$ and $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$ is divergent.

D. It is divergent because $0 \le \dfrac{1}{x^2} \le \dfrac{2+\sin(x)}{x^2}$ and $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ is divergent.

E. None of the above; the comparison theorem for improper integrals cannot be applied to this integral.

16. (4 points) Evaluate $\displaystyle\lim_{x\to\infty} \frac{\ln(3x^2)}{\ln(5x+1)}$.

A. $\dfrac{3}{5}$ B. $\dfrac{6}{5}$ C. 2 D. $\dfrac{\ln(6)}{\ln(5)}$ E. The limit does not exist

17. (4 points) The rate of decay of a radioactive material is proportional to the amount of that material present. If it takes 5 years for one third of the material to decay, how many years does it take for half of the material to decay?

A. $\dfrac{5\ln\left(\frac{2}{3}\right)}{\ln\left(\frac{1}{2}\right)}$ B. $\dfrac{\ln\left(\frac{2}{3}\right)}{5}$ C. $\dfrac{\ln\left(\frac{2}{3}\right)}{5\ln\left(\frac{1}{2}\right)}$ D. $\dfrac{5\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{2}{3}\right)}$ E. $5\ln\left(\frac{3}{4}\right)$

More Challenging Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

18. (8 points) Find the integral $\displaystyle\int \left(\sqrt{\sin(2x)} \;-\; \cos(2x)\right)^2 \, dx$

19. (6 points) Prove that the curves $y = \dfrac{\pi}{6} - \sin^{-1}(\dfrac{x}{2})$ and $y = \dfrac{\sqrt{3}}{2}\left(e^{x^2-1} - 1\right)$ intersect at right angles at the point $(1, 0)$.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET PAGE 1

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\int \frac{1}{x}\ dx = \ln|x| + C$
- $\int \tan x\ dx = \ln|\sec x| + C$
- $\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\int a^x\ dx = \frac{a^x}{\ln a} + C$ for $a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

Derivatives

- $\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$