Name: ______________________________

Section: ______________ **Recitation Instructor**: ______________________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 12.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

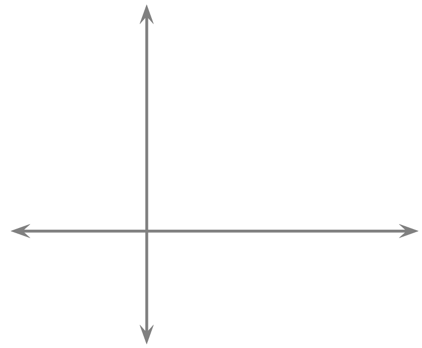
1. (5 points) Let $f(x,y) = x\,e^{y-2} + y^2 \ln x$. Find ∇f at $P(1,2)$.

2. (9 points) Let $g(x,y) = 8x^3 - 12xy + y^2$. Find and classify each critical point of g as a local minimum, a local maximum, or a saddle point.

3. Consider the integral below and answer the questions that follow.

$$\int_0^1 \int_{\pi x}^{\pi} \sin(y^2/\pi)\, dy\, dx$$

(a) (3 points) Sketch the region of integration.

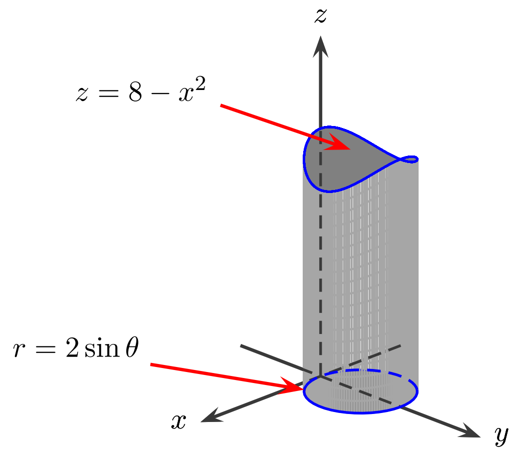


(b) (5 points) Evaluate the integral above by reversing the order of integration.

4. (6 points) Evaluate the integral below.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{1}{\sqrt{5+x^2+y^2}}\, dy\, dx$$

5. (6 points) Set up but **Do Not Evaluate** the iterated integral for computing the volume of a region D if D is the right circular cylinder whose base is the disk $r = 2\sin\theta$ (in the xy-plane) and whose top lies on the surface $z = 8 - x^2$.



6. (8 points) Find the area of the surface of the cap cut from the paraboloid $z = 12 - x^2 - y^2$ by the cone $z = \sqrt{x^2 + y^2}$.

7. Let $\mathbf{F} = 2xy\,\mathbf{i} + (x^2 - \cos z)\,\mathbf{j} + y\sin z\,\mathbf{k}$ and answer the questions below.

   (a) (7 points) Find a function f so that $\nabla f = \mathbf{F}$.

   (b) (4 points) Let C be any path from $(-2, 1, \pi/2)$ to $(1, 3, \pi)$. Evaluate the integral

$$\int_C 2xy\,dx + (x^2 - \cos z)\,dy + y\sin z\,dz$$

   (c) (3 points) Calculate $\operatorname{curl}\mathbf{F}$.

8. (7 points) Let E be the portion of the ball $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant: $x \geq 0$, $y \geq 0$, $z \geq 0$.

   Rewrite the triple integral below as an iterated integral in spherical coordinates and evaluate.

$$\iiint_E 5y\, dV$$

9. (7 points) Let $\mathbf{F}(x, y) = \left\langle y^2, x^2 \right\rangle$. Use Green's Theorem to calculate the work done by $\mathbf{F}$ on a particle moving counter-clockwise around the triangle bounded by $x = 0,\ y = 0,\ x + y = 1$.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

10. (4 points) Let $P = P(1/2, 2)$ and $\mathbf{u} = \dfrac{1}{\sqrt{2}}\,\mathbf{i} - \dfrac{1}{\sqrt{2}}\,\mathbf{j}$. If $\nabla f(P) = 3\,\mathbf{i} + 2\,\mathbf{j}$, then $D_{\mathbf{u}}(P) =$

A. $\dfrac{11}{2}$

B. $\dfrac{-3}{2\sqrt{2}}$

C. $\dfrac{3}{\sqrt{2}}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$

D. $\dfrac{1}{\sqrt{2}}$

E. None of the above

11. (4 points) Suppose that the line segment C is given by the parametrization: $\mathbf{r}(t) = (t+1)\,\mathbf{i} + 2t\,\mathbf{j},\ 0 \le t \le 3$. Evaluate the integral below.

$$\int_C \langle x^2, -y \rangle \cdot d\mathbf{r}$$

A. 57

B. 39

C. 21

D. 3

E. None of the above

12. (4 points)

$$\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z}\, dy\, dz\, dx =$$

A. $\dfrac{\sin 8}{4}$

B. $\dfrac{-\cos 8}{2}$

C. $\dfrac{-\cos 8}{4}$

D. $\dfrac{-\cos 4}{2}$

E. None of the above

13. (4 points) Parameterize the part of the plane $x + z = 3$ that lies above the disk $(x-1)^2 + y^2 \le 1$.

A. $\mathbf{r}(s,t) = \langle s, t, 3-s \rangle$, with $s \in [0, 2\cos t]$, and $t \in [-1, 1]$.

B. $\mathbf{r}(s,t) = \langle s, t, 3-s \rangle$ with $s \in [0, 2]$ and $t \in [-1, 1]$.

C. $\mathbf{r}(s,t) = \langle s\cos t, s\sin t, 3 - s\cos t \rangle$ with $s \in [0, 2]$ and $t \in [-\pi/2, \pi/2]$.

D. $\mathbf{r}(s,t) = \langle s\cos t, s\sin t, 3 - s\cos t \rangle$ with $s \in [0, 2\cos t]$ and $t \in [-\pi/2, \pi/2]$.

E. None of the above

14. (4 points) Let $\mathbf{F}(x,y) = \nabla \arctan\left(\dfrac{y}{x}\right) = \dfrac{-y}{x^2+y^2}\,\mathbf{i} + \dfrac{x}{x^2+y^2}\,\mathbf{j}$.

If C is the unit circle parameterized by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$, $0 \le t \le 2\pi$, then $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} =$

A. 0

B. π

C. 2π

D. Undefined

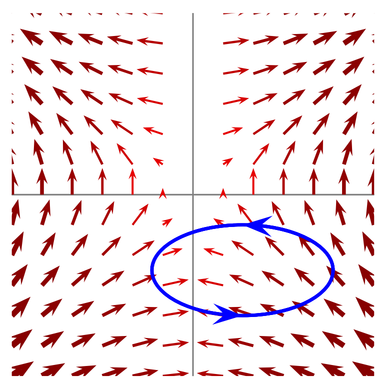
E. None of the above

15. (4 points) Let f be a differentiable function of x and y with continuous second order partial derivatives. Let $\mathbf{F} = \nabla f = M\,\mathbf{i} + N\,\mathbf{j}$ be a gradient field and let C be the (positively oriented) ellipse as shown in the sketch below. Consider the following statements.

(a) $\mathbf{F}$ is a conservative vector field.

(b) $\displaystyle\oint_C M\,dx + N\,dy = 0$

(c) $(\operatorname{curl}\mathbf{F}) \cdot \mathbf{k} > 0$



A. All three statements are true.

B. Only (a) and (b) are true.

C. Only (a) and (c) are true.

D. Only (b) and (c) are true.

E. None of the above

16. (4 points) Let $\mathbf{F} = \langle x^2 y, -yz, z^2 \rangle$. Which of the following is true?

A. $\operatorname{curl} \mathbf{F} = 2xy + z$ and $\operatorname{div} \mathbf{F} = y - x^2$

B. $\operatorname{curl} \mathbf{F} = \langle y, 0, -x^2 \rangle$ and $\operatorname{div} \mathbf{F} = 2xy + z$

C. $\operatorname{curl} \mathbf{F} = y - x^2$ and $\operatorname{div} \mathbf{F} = 2xy + z$

D. $\operatorname{curl} \mathbf{F} = \langle 2xy, z, 0 \rangle$ and $\operatorname{div} \mathbf{F} = \langle y, 0, -x^2 \rangle$

E. None of the above

17. (4 points) Identify the surface given by the vector equations $\mathbf{r}(u, v) = \langle u, 5 \sin 3v, 4 \cos 3v \rangle$.

A. plane

B. elliptic paraboloid

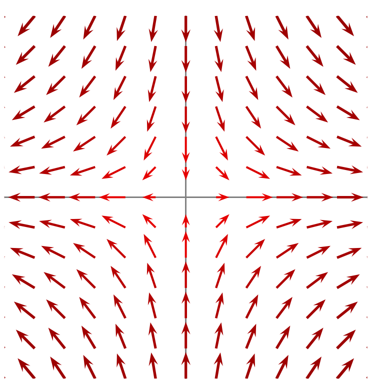
C. cylinder

D. ellipsoid

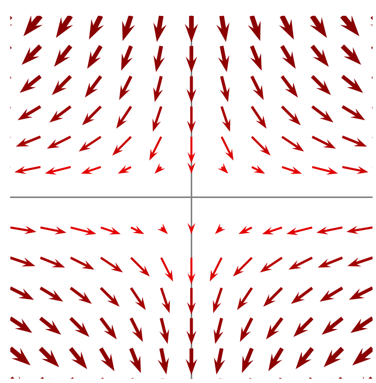
E. None of the above

18. (4 points) Which of the following vector field plots could be $\mathbf{F} = xy^2\,\mathbf{i} - x\,\mathbf{j}$?

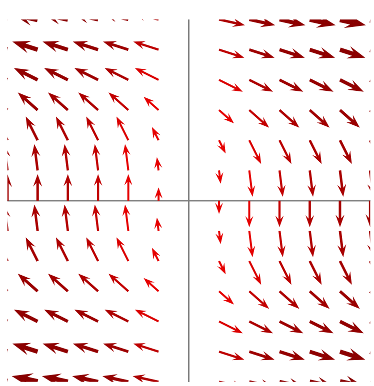
A.



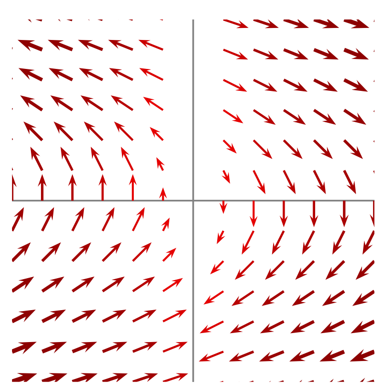
B.



C.



D.



E. None of the above

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	14	
7	12	
8	12	
9	12	
Total:	106	

Vectors in Space

Suppose $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$:

- **Unit Vectors**:
$$\mathbf{i} = \langle 1, 0, 0 \rangle$$
$$\mathbf{j} = \langle 0, 1, 0 \rangle$$
$$\mathbf{k} = \langle 0, 0, 1 \rangle$$
- **Length** of vector $\mathbf{u}$
$$|\mathbf{u}| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$
- **Dot Product**:
$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 = |\mathbf{u}||\mathbf{v}| \cos\theta$$
- **Cross Product**:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
- **Vector Projection**: $\text{proj}_{\mathbf{u}} \mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2} \mathbf{u}$

Partial Derivatives

- **Chain Rule**: Suppose $z = f(x, y)$ and $x = g(t)$ and $y = h(t)$ are all differentiable then
$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Curves and Planes in Space

- **Line** parallel to $\mathbf{v}$: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$
- **Plane** normal to $\mathbf{n} = \langle a, b, c \rangle$:
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
- **Arc Length** of curve $\mathbf{r}(t)$ for $t \in [a, b]$.
$$L = \int_a^b |\mathbf{r}'(t)| \, dt$$
- **Unit Tangent Vector** of curve $\mathbf{r}(t)$
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

More on Surfaces

- **Directional Derivative:** $D_{\mathbf{u}} f(x, y) = \nabla f \cdot \mathbf{u}$
- **Second Derivative Test** Suppose $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let
$$D = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$
  (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
  (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
  (c) If $D < 0$ then $f(a, b)$ is a saddle point.

Geometry / Trigonometry

- Area of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $A = \pi ab$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$

Multiple Integrals

- **Area**: $A(D) = \iint_D 1 \; dA$
- **Volume**: $V(E) = \iiint_E 1 \; dV$

Polar/Cylindrical

- Transformations

$$r^2 = x^2 + y^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$y/x = \tan\theta$$

- $\iint_D f(x,y)\; dA = \iint_D f(r\cos\theta, r\sin\theta)\; r\; dr\; d\theta$
- $\iiint_E f(x,y,z)\; dV = \iiint_E f(r\cos\theta, r\sin\theta, z)\; r\; dz\; dr\; d\theta$

Spherical

- Transformations

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$
$$\rho^2 = x^2 + y^2 + z^2$$

- $\iint_E f(x,y,z)\; dV =$
$\iint_E f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)(\rho^2\sin\phi)\; d\rho\; d\phi\; d\theta$

Additional Definitions

- $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$
- $\text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F}$
- $\mathbf{F}$ is conservative if $\text{curl}(\mathbf{F}) = 0$

Line Integrals

- **Fundamental Theorem of Line Integrals**

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

- **Green's Theorem**

$$\int_C P\; dx + Q\; dy = \iint_D (Q_x - P_y)\, dA$$