Name: _____

Section: _____   **Recitation Instructor**: _____

INSTRUCTIONS

- Fill in your name, etc. on this first page.

- Without fully opening the exam, check that you have pages 1 through 11.

- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.

- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.

- If you have any questions please raise your hand and a proctor will come to you.

- You will be given exactly 90 minutes for this exam.

- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.

- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).

- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.

- No scratch paper is permitted. If you need more room use the back of a page.

- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the
above instructions and statements
regarding academic honesty: _____

                                    SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. (7 points) Find a parametrization of the line of intersection between planes $2x + z = 1$ and $y + z = 2$.

2. (7 points) What is the length of the curve $\mathbf{r}(t) = \langle \sin(2t), t, \cos(2t) \rangle$ from $t = 0$ to $t = \pi$?

3. (7 points) Evaluate the limit or show that it does not exist: $\displaystyle \lim_{\substack{x \to 1 \\ y \to -1}} \frac{\sqrt{x} - \sqrt{2 + y}}{x - y - 2}$

4. (7 points) Evaluate the limit or show that it does not exist: $\displaystyle \lim_{\substack{x \to 0^+ \\ y \to 0^+}} \frac{4x^2 + 9y^2}{2xy}$

5. Consider the function $f(x, y) = \sqrt{x^2 + 6xy + 9y^2}$ in the first quadrant.
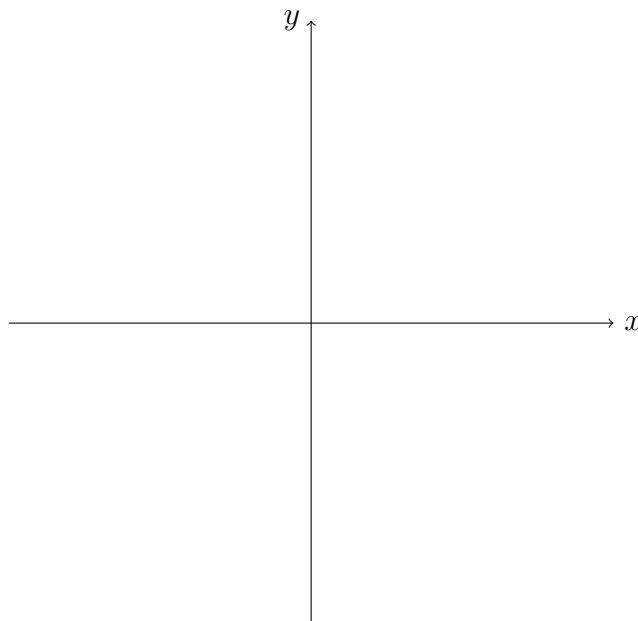
    (a) (5 points) Calculate the partial derivatives, f_x and f_y.

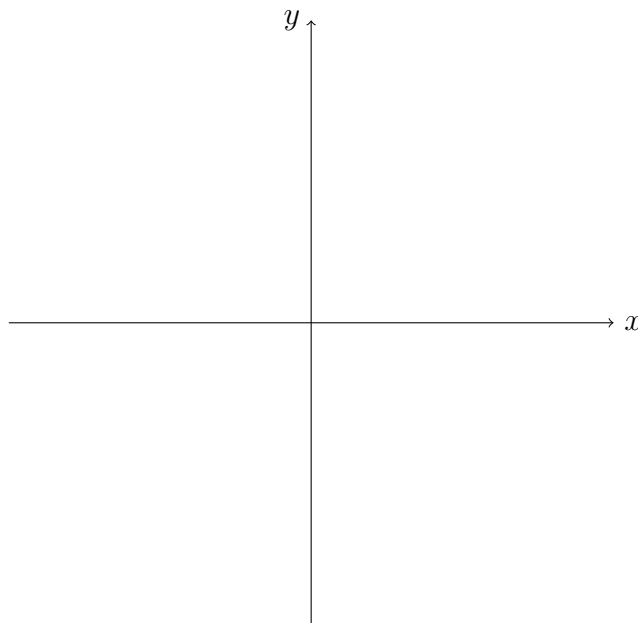   (b) (5 points) Use your answer in part (a) to find the linearization of f at $(1, 2)$.

    (c) (4 points) Use your answer in part (b) to approximate $f(0.7, 2.1)$.

6. Consider the function $f(x, y) = 2 - \sqrt{y^2 + x - 1}$

  (a) (5 points) Sketch the domain of f.

  (b) (6 points) Sketch the level curves of f for levels $k = 0$, $k = 1$, and $k = 2$.

  (c) (3 points) What is the range of f?

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

7. (4 points) The intersection of the quadric surfaces defined by $x^2 + y^2 + z^2 = 4$ and $x^2 + z^2 = 4$ is:

    A. one point

    B. two points

    C. a circle

    D. a straight line

    E. a parabola

8. (4 points) Find the equation for the sphere in standard form centered at $(1, 2, 3)$ with radius 3.

    A. $(x + 1)^2 + (y + 2)^2 + (z + 3)^2 = 9$

    B. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 8$

    C. $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 9$

    D. $(x - 1) + (y - 2) + (z - 3) = 3$

    E. $x + 2y + 3z = 3$

9. (4 points) What is the positive value of parameter t corresponding to a point where the line defined by $\langle 4t, 3t, 1 \rangle$ intersects the cone $z^2 = x^2 + y^2$?

    A. $5/2$

    B. $3/10$

    C. $8/5$

    D. $1/5$

    E. 15

10. (4 points) Find the area of the triangle formed by the points $O(0,0,0)$, $P(1,0,2)$ and $Q(2,-1,-1)$

     A. 4

     B. $\sqrt{15/2}$

     C. $\sqrt{11}$

     D. $\sqrt{10}$

     E. 3

11. (4 points) Which of the following is a parametrization for the full curve of intersection between
$x^2 + y^2 - z^2 = 0$ and $z = x + 1$

     A. $\mathbf{r}(t) = \left\langle t, \sqrt{2t+1}, t+1 \right\rangle$ with $t \in [-1/2, \infty)$

     B. $\mathbf{r}(t) = \left\langle \frac{t^2-1}{2}, t, \frac{t^2+1}{2} \right\rangle$ with $t \in [0, \infty)$

     C. $\mathbf{r}(t) = \left\langle \frac{t^2-1}{2}, t, \frac{t^2+1}{2} \right\rangle$ with $t \in (-\infty, \infty)$

     D. $\mathbf{r}(t) = \langle \cos(t), \sin(t), -2 \rangle$ with $t \in [0, 2\pi]$

     E. $\mathbf{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$ with $t \in [0, 2\pi]$

12. (4 points) Which of the following is the unit tangent vector to the curve: $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{2}{3}t^3 \right\rangle$
at $\mathbf{r}(1) = \langle 2, 1, 2/3 \rangle$

     A. $\mathbf{r}(t) = \langle 2, 1, 2/3 \rangle$

     B. $\mathbf{r}(t) = \langle 2, 2, 2 \rangle$

     C. $\mathbf{r}(t) = \dfrac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle$

     D. $\mathbf{r}(t) = \langle 2, 2t, 2t^2 \rangle$

     E. $\mathbf{r}(t) = \langle 2, 0, -\cos(t) \rangle$

13. (4 points) Consider the function $f(x, y) = \ln(y)x$. Calculate $f_{xx} + f_{xy}$.

    A. $f_{xx} + f_{xy} = \dfrac{1}{y} + 1$

    B. $f_{xx} + f_{xy} = \dfrac{1}{y}$

    C. $f_{xx} + f_{xy} = 0$

    D. $f_{xx} + f_{xy} = 1$

    E. $f_{xx} + f_{xy} = \dfrac{1}{x^2 y^2} - \dfrac{2}{x^3 y}$

14. (4 points) Suppose $f(x, y) = x^2 y$ where $x(t) = 3\cos(t) + 2\sin(t)$ and $y(t) = 5\cos(t) - 4\sin(t)$. Which of the following is equal to $\dfrac{df}{dt}$ at $t = 0$?

    A. 6

    B. 24

    C. -14

    D. -15

    E. -3

15. (4 points) Find the vector $\mathbf{a}$ with $|\mathbf{a}| = 6$ orthogonal to the plane given by $x + 2y + 2z + 5 = 0$.

    A. $\langle 0, 3, -1 \rangle$

    B. $\left\langle 0, \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle$

    C. $\langle 2, 0, t^2 \rangle$

    D. $\langle 1, 2, 2 \rangle$

    E. $\langle 2, 4, 4 \rangle$

More Challenging Problem(s). Show all work to receive credit.

16. (7 points) Find the distance between the two skew lines given by vector equations

$$\langle 1 + 2t, 0, t \rangle \qquad \text{and} \qquad \langle -1 + t, 2 + t, 1 + t \rangle$$

17. (7 points) An object moves along the surface $x^2 + z^2 = 9$ in $\mathbb{R}^3$ via the parametric equations:

$$x(t) = 3\cos(\pi t) \qquad \text{and} \qquad y(t) = 4\pi t$$

What is the speed and of the object at time $t = 2$?

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Vectors in Space

Suppose $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$:

- **Unit Vectors**:
$$\mathbf{i} = \langle 1, 0, 0 \rangle$$
$$\mathbf{j} = \langle 0, 1, 0 \rangle$$
$$\mathbf{k} = \langle 0, 0, 1 \rangle$$

- **Length** of vector $\mathbf{u}$
$$|\mathbf{u}| = \sqrt{u_1{}^2 + u_2{}^2 + u_3{}^2}$$

- **Dot Product**:
$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$
$$= |\mathbf{u}||\mathbf{v}| \cos \theta$$

- **Cross Product**:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- **Vector Projection**: $\quad \mathrm{proj}_{\mathbf{u}} \mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2} \mathbf{u}$

Partial Derivatives

- **Chain Rule**: Suppose $z = f(x, y)$ and $x = g(t)$ and $y = h(t)$ are all differentiable then
$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Curves and Planes in Space

- **Line** parallel to $\mathbf{v}$: $\quad \mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$

- **Plane** normal to $\mathbf{n} = \langle a, b, c \rangle$:
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

- **Arc Length** of curve $\mathbf{r}(t)$ for $t \in [a, b]$.
$$L = \int_a^b |\mathbf{r}'(t)|\, dt$$

- **Unit Tangent Vector** of curve $\mathbf{r}(t)$
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

Trigonometry

- $\sin^2 x + \cos^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$