

Name: _____

Section: _____ Recitation Instructor: _____

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 12.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page for scratch work. **Work on the back of pages will not be graded.**
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the
above instructions and statements
regarding academic honesty: _____

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. (14 points) Consider the function $f(x, y) = 2x^2 - 4xy + y^4 + 2$

(a) Find the critical points of f and classify them as local minima, local maxima, or saddle points.

(b) Let C be the curve $y = x$ for $x \in [0, 3]$. What is the absolute minimum and maximum of f on C ?

2. (14 points) Evaluate

$$\int_C ((x + y)\mathbf{i} + (y - z)\mathbf{j} + z^2\mathbf{k}) \cdot d\mathbf{r}$$

where C is given by vector function $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^2\mathbf{k}$, $0 \leq t \leq 1$.

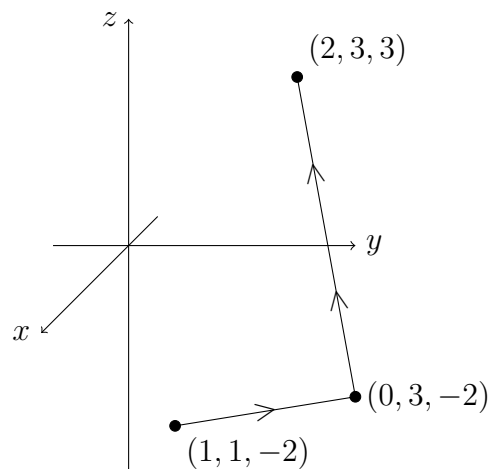
3. (14 points) Find the surface area of the surface given by the graph $z = 1 + 3x + \ln(\sqrt{y}) - \frac{5}{2}y^2$ over the domain $1 \leq x \leq 4$, $1 \leq y \leq 2$.

4. (14 points) Consider the vector field $\mathbf{F}(x, y, z) = (2xy^2z^3) \mathbf{i} + (2x^2yz^3) \mathbf{j} + (3x^2y^2z^2) \mathbf{k}$

(a) Compute the $\text{curl}(\mathbf{F})$ (show your calculations)

(b) Find a function f such that $\mathbf{F} = \nabla f$.

(c) Evaluate a line integral $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$ where C is the curve shown below.



Multiple Choice. Circle the best answer. No work needed. No partial credit available.

5. (4 points) E is the cube given by $0 \leq x \leq a$, $0 \leq y \leq a$, $0 \leq z \leq a$ with mass density $\sigma(x, y, z) = x^2 + y^2 + z^2$. The mass is given by the formula:

- A. $\int_0^a \int_0^a \int_0^{\sqrt{1-x^2-y^2}} 1 \, dz \, dy \, dx$
- B. $\int_0^a \int_0^a \int_0^{\sqrt{1-x^2-y^2}} x^2 + y^2 + z^2 \, dz \, dy \, dx$
- C. $\int_0^a \int_0^a \int_0^a 1 \, dz \, dy \, dx$
- D. $\int_0^a \int_0^a \int_0^a x^2 + y^2 + z^2 \, dz \, dy \, dx$
- E. None of the above.

6. (4 points) Which of the following is the equations of the tangent plane to the surface $xyz^2 = 6$ at the point $(3, 2, 1)$.

- A. $x + y = 3$
- B. $2x + 3y + 12z = 0$
- C. $2x - 3y + 6z = 24$
- D. $2x + 3y + 12z = 24$
- E. $y + 3z = 24$

7. (4 points) Evaluate $\iiint_B \frac{1}{\sqrt{x^2 + y^2}} \, dA$, where B lies between the spheres

$$x^2 + y^2 + z^2 = 4 \text{ and } x^2 + y^2 + z^2 = 9$$

- A. 3π
- B. $-\pi^2$
- C. $5\pi^2$
- D. π
- E. π^2

8. (4 points) Reversing the order of integration $\int_2^4 \int_1^{9-2y} 3x + y \, dx \, dy =$

A. $\int_2^6 \int_5^{y-1} 3x + y \, dy \, dx$

B. $\int_1^5 \int_2^{(9-x)/2} 3x + y \, dy \, dx$

C. $\int_1^5 \int_5^{2-x} 3x + y \, dy \, dx$

D. $\int_2^4 \int_1^5 3x + y \, dy \, dx$

E. $\int_1^{9-2y} \int_2^4 3x + y \, dy \, dx$

9. (4 points) If $\mathbf{F}(x, y, z) = z\mathbf{i} + (2x + z)\mathbf{j} + 3z\mathbf{k}$, then $\text{div}(\mathbf{F}) =$

A. 0

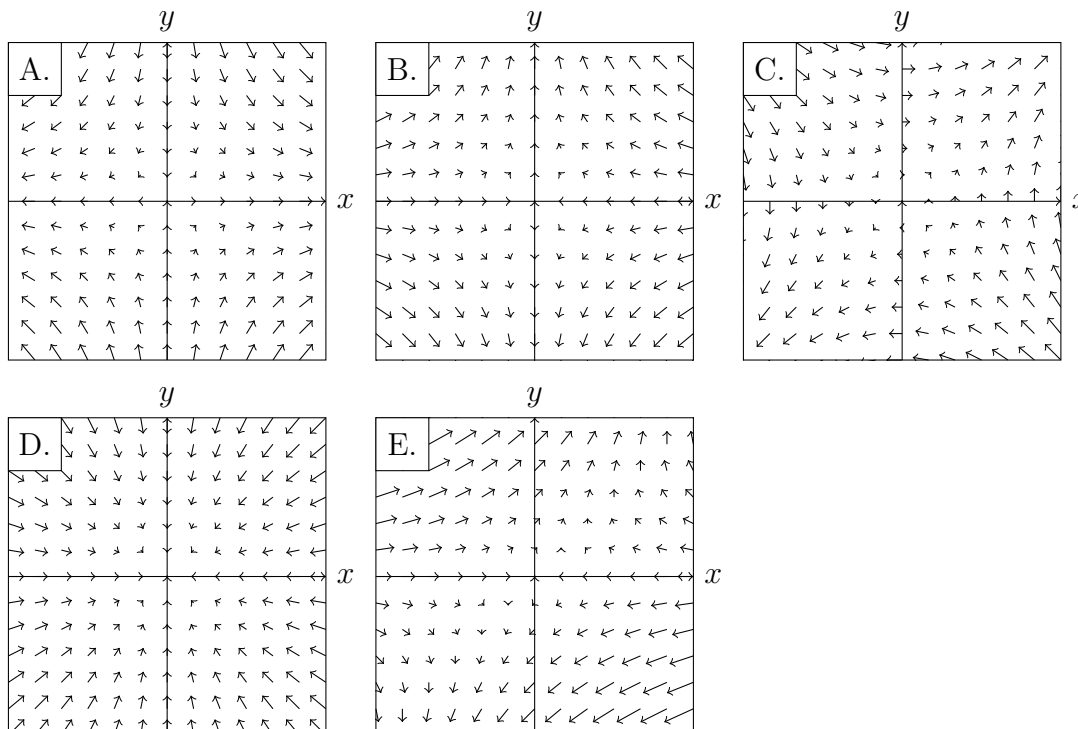
B. $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$

C. $1 + x + y$

D. xz

E. $x^2 + y + 2z$

10. (4 points) Which of the following could be the gradient vector field ∇f of the function $f(x, y) = xy$.



11. (4 points) Evaluate $\int_C \langle -2y, y \rangle \cdot d\mathbf{r}$, where C is the line segment from $(0, 0)$ to $(1, 2)$.
- A. 3
 - B. 4
 - C. 5
 - D. 0
 - E. 9
12. (4 points) The area of the surface with parametric equations $x = u^2$, $y = uv$, $z = \frac{1}{2}v^2$, with $0 \leq u \leq 1$ and $0 \leq v \leq 2$
- A. 1
 - B. 2
 - C. 3
 - D. 4
 - E. 5
13. (4 points) Consider the closed curve C which is the union of three paths; the straight line from $(0, 0)$ to $(1, 0)$, the arc of the unit circle from $(1, 0)$ to $(0, 1)$, and finally, straight line from $(0, 1)$ to $(0, 0)$. Evaluate $\int_C (x + y)dx - (x - y)dy$
- A. -2
 - B. 3
 - C. $-\pi/2$
 - D. $-\pi/4$
 - E. $-\pi^2$

More Challenging Problem(s). Show all work to receive credit.

14. (8 points) Consider the vector field $\mathbf{F} = \frac{x-y}{x^2+y^2}\mathbf{i} + \frac{Ax+y}{x^2+y^2}\mathbf{j}$

(a) Compute the value of A so that the vector field is conservative on its domain.

(b) For this value of A , explain the equality $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_S \mathbf{F} \cdot d\mathbf{r}$ where C is the unit circle centered at the origin and S is the boundary of the square $[-1, 1] \times [-1, 1]$.

15. (6 points) Let the vector field $\mathbf{F}(x, y)$ in the plane satisfies relation $\mathbf{F}(x, y) = \mathbf{F}(-x, -y)$.

Show that the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ where C is the boundary of the square $[-1, 1] \times [-1, 1]$.

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are **BOXED**.

When you are completely happy with your work please bring your exam to the front to be handed in.

Please have your MSU student ID ready so that it can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET PAGE 1

Vectors in Space

Suppose $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$:

- **Unit Vectors:**

$$\mathbf{i} = \langle 1, 0, 0 \rangle$$

$$\mathbf{j} = \langle 0, 1, 0 \rangle$$

$$\mathbf{k} = \langle 0, 0, 1 \rangle$$

- **Length of vector \mathbf{u}**

$$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

- **Dot Product:**

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$$

$$= |\mathbf{u}||\mathbf{v}| \cos \theta$$

- **Cross Product:**

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- **Vector Projection:** $\text{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2} \mathbf{u}$

Partial Derivatives

- **Chain Rule:** Suppose $z = f(x, y)$ and $x = g(t)$ and $y = h(t)$ are all differentiable then

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

Curves and Planes in Space

- **Line parallel to \mathbf{v} :** $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$

- **Plane normal to $\mathbf{n} = \langle a, b, c \rangle$:**

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

- **Arc Length of curve $\mathbf{r}(t)$ for $t \in [a, b]$.**

$$L = \int_a^b |\mathbf{r}'(t)| dt$$

- **Unit Tangent Vector of curve $\mathbf{r}(t)$**

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

More on Surfaces

- **Directional Derivative:** $D_{\mathbf{u}}f(x, y) = \nabla f \cdot \mathbf{u}$

- **Second Derivative Test** Suppose

$$f_x(a, b) = 0 \text{ and } f_y(a, b) = 0. \text{ Let}$$

$$D = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
- If $D < 0$ then $f(a, b)$ is a saddle point.

Geometry / Trigonometry

- Area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $A = \pi ab$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$

FORMULA SHEET PAGE 2

Multiple Integrals

- **Area:** $A(D) = \iint_D 1 \, dA$
- **Volume:** $V(E) = \iiint_E 1 \, dV$

Polar/Cylindrical

- Transformations

$$\begin{aligned} r^2 &= x^2 + y^2 \\ x &= r \cos \theta \\ y &= r \sin \theta \\ y/x &= \tan \theta \end{aligned}$$

- $\iint_D f(x, y) \, dA = \iint_D f(r \cos \theta, r \sin \theta) r \, dr \, d\theta$
- $\iiint_E f(x, y, z) \, dV = \iiint_E f(r \cos \theta, r \sin \theta, z) r \, dz \, dr \, d\theta$

Spherical

- Transformations

$$\begin{aligned} x &= \rho \sin \phi \cos \theta \\ y &= \rho \sin \phi \sin \theta \\ z &= \rho \cos \phi \\ \rho^2 &= x^2 + y^2 + z^2 \end{aligned}$$

- $\iiint_E f(x, y, z) \, dV = \iiint_E f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) (\rho^2 \sin \phi) \, d\rho \, d\phi \, d\theta$

Additional Definitions

- $\text{curl}(\mathbf{F}) = \nabla \times \mathbf{F}$
- $\text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F}$
- \mathbf{F} is conservative if $\text{curl}(\mathbf{F}) = 0$

Line Integrals

- **Fundamental Theorem of Line Integrals**

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

- **Green's Theorem**

$$\int_C P \, dx + Q \, dy = \iint_D (Q_x - P_y) \, dA$$