Name: ______________________________

Section: ______________ **Recitation Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

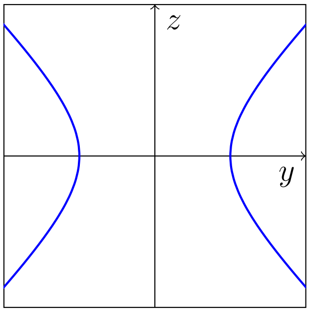
- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page for scratch work. Work on the back of pages will typically not be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

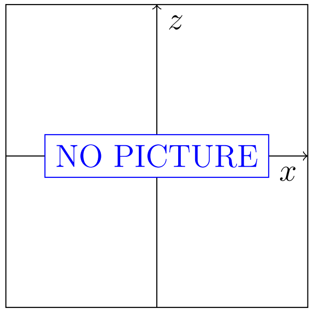
SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

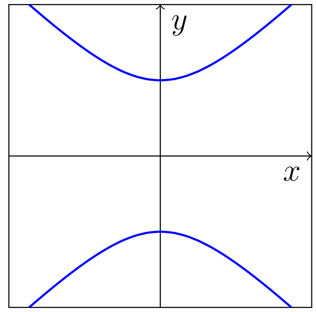
1. (7 points) Sketch the traces of the surface $-x^2 + y^2 - z^2 = 1$ in coordinate planes and the surface itself. Determine the type of surface by circling one of the choices.



yz−plane



xz−plane



xy−plane

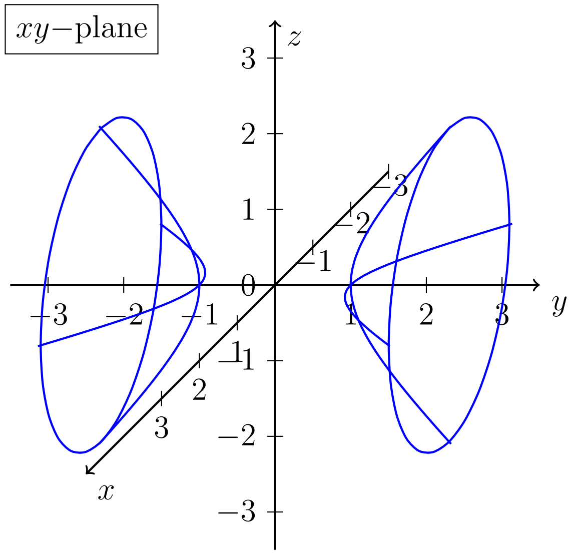
A. elliptic paraboloid

B. hyperbolic paraboloid

C. hyperboloid of one sheet

D. hyperboloid of two sheets

E. cone



2. (7 points) Find the distance between line $\mathbf{r}(t) = \langle 1, 3+t, 4+t \rangle$ and the line passing through points $(-1, 2, 5)$ and $(1, 2, 5)$.

Solution:

$$\mathbf{v}_1 = \langle 0, 1, 1 \rangle$$
$$\mathbf{v}_2 = \langle 2, 0, 0 \rangle$$
$$\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 0, 2, -2 \rangle$$

So consider $P = (1, 3, 4)$ and $R = (-1, 2, 5)$ which are one the first and second line respectively then $\overrightarrow{PR} = \langle -2, -1, 1 \rangle$ joins the two lines and the distance between them is

$$d = |\text{comp}_{\mathbf{n}}(\overrightarrow{PR})| = \frac{|\overrightarrow{PR} \cdot \mathbf{n}|}{||\mathbf{n}||}$$
$$= \frac{|-2-2|}{\sqrt{4+4}} = \sqrt{2}$$

3. Let $f(x, y, z) = x^2 - xy + 3\sin z$ and answer the following questions

(a) (5 points) Calculate the partial derivatives f_x, f_y and f_z.

Solution: $\boxed{f_x = 2x - y,\ f_y = -x,\ f_z = 3\cos z}$.

(b) (5 points) Use your answer in part (a) to find the linearization of f at $(2, 1, 0)$.

Solution:

$$f(2,1,0) = 2$$
$$f_x(2,1,0) = 3$$
$$f_y(2,1,0) = -2$$
$$f_z(2,1,0) = 3$$

Linearization $L(x, y, z) = 2 + 3(x - 2) - 2(y - 1) + 3z = \boxed{3x - 2y + 3z - 2}$.

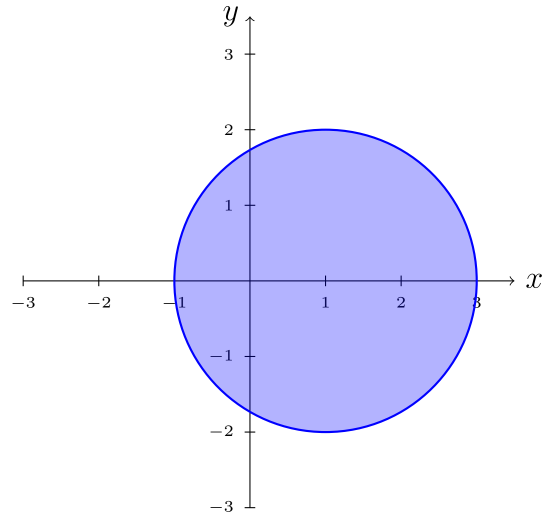
(c) (4 points) Use your answer in part (b) to approximate the number $f(2.2, 0.9, -0.1)$.

Solution:

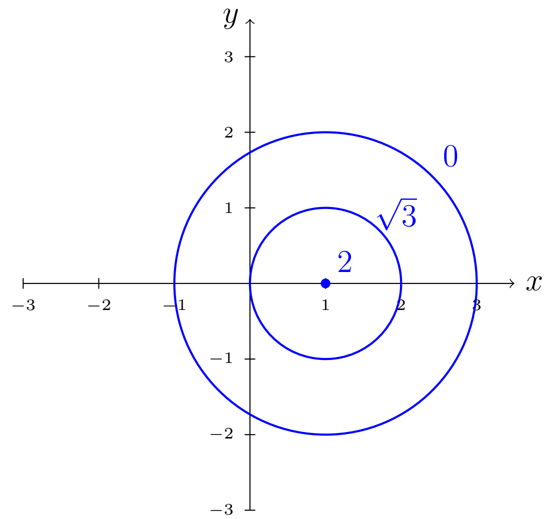
$$L(2.2, 0.9, -0.1) = 2 + 3(0.2) - 2(-0.1) + 3(-0.1) = 2 + 0.1 = \boxed{2.5}$$

4. Consider the function $f(x, y) = \sqrt{3 + 2x - x^2 - y^2}$

(a) (5 points) Sketch the domain of f.



(b) (6 points) Sketch the level curves of f for levels $k = 0, \sqrt{3}, 2$.



(c) (3 points) What is the range of f?

Solution: $[0, 2]$

5. (7 points) Evaluate the limit: $\lim\limits_{\substack{(x,y)\to(1,0)\\ y\neq 0}} \dfrac{\sqrt{x-y}-\sqrt{x+y}}{y}$.

Solution:

$$\begin{aligned}
\lim_{\substack{(x,y)\to(1,0)\\ y\neq 0}} \frac{\sqrt{x-y}-\sqrt{x+y}}{y} &= \lim_{\substack{(x,y)\to(1,0)\\ y\neq 0}} \frac{\sqrt{x-y}-\sqrt{x+y}}{y}\cdot\left(\frac{\sqrt{x-y}+\sqrt{x+y}}{\sqrt{x-y}+\sqrt{x+y}}\right)\\
&= \lim_{\substack{(x,y)\to(1,0)\\ y\neq 0}} \frac{x-y-(x+y)}{y}\cdot\left(\frac{1}{\sqrt{x-y}+\sqrt{x+y}}\right)\\
&= \lim_{\substack{(x,y)\to(1,0)\\ y\neq 0}} \frac{-2}{1}\cdot\left(\frac{1}{\sqrt{x-y}+\sqrt{x+y}}\right)\\
&= \frac{-2}{1}\cdot\left(\frac{1}{\sqrt{1}+\sqrt{1}}\right) = \boxed{-1}
\end{aligned}$$

6. (7 points) Evaluate $\lim\limits_{(x,y)\to(1,0)} \dfrac{2(x-1)y}{x^2-2x+1-y^2}$ or show that the limit does not exist .

Solution:

Path 1: $y=0$, $x\to 1$

$$\lim_{x\to 1}\frac{2(x-1)(0)}{x^2-2x+1-(0)} = \lim_{x\to 1}\frac{0}{x^2-2x+1} = 0$$

Path 2: $y=2x-2$, $x\to 1$

$$\lim_{x\to 1}\frac{2(x-1)(2x-2)}{x^2-2x+1-(2x-2)^2} = \lim_{x\to 1}\frac{4(x-1)^2}{-3(x-1)^2} = -4/3$$

Since the limit has different results along different paths the limit $\boxed{DNE}$.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

7. (4 points) Find the center and the radius of sphere $x^2 + y^2 + z^2 - 4x + 2y - 4z = 0$.

   A. center at $(2, 1, 2)$ radius $r = 3$

   B. center at $(2, 1, 2)$ radius $r = 2$

   C. center at $(2, -1, 2)$ radius $r = 3$

   D. center at $(2, -1, 2)$ radius $r = 2$

   E. None of the above

8. (4 points) The position of the particle moving in space at time $t \geq 0$ is

$$\mathbf{r}(t) = (2 + 2\cos(t))\mathbf{i} - 2\sin(t)\mathbf{j} + (3 - \frac{t}{\pi})\mathbf{k}.$$

   Find the first time moment t_0 such that the velocity vector $\mathbf{v}(t_0)$ is orthogonal to the vector $\mathbf{i} - \mathbf{j}$.

   A. $\pi/6$

   B. $\pi/4$

   C. $\pi/3$

   D. $\pi/2$

   E. None of the above

9. (4 points) The trajectory of a flying saucer is given by the vector function

$$\mathbf{r}(t) = (5\sin(t))\mathbf{i} + (5\cos(t))\mathbf{j} + 12t\mathbf{k} \quad \text{for} \quad t \geq 0$$

   where coordinate is measured in miles and time in minutes. Find the time moment t_0 when the flying saucer travels exactly the distance 26 miles along the curve from the starting point.

   A. 3π

   B. 1

   C. 2

   D. 3

   E. None of the above

10. (4 points) Find the acute angle between two planes $x + y = 1$ and $y + z = 1$.

A. $\pi/6$

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

E. None of the above

11. (4 points) Find the value of $\partial z/\partial x$ at the point $(1, 1, 1)$ if z as a function of x and y is defined by the equation $xy + z^3x - 2yz = 0$.

A. -2

B. -1

C. 1

D. 2

E. None of the above

12. (4 points) Evaluate $\dfrac{dw}{dt}$ at $t = 3$ if $w = \dfrac{x}{z} + \dfrac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = \dfrac{1}{t}$.

A. -1

B. 0

C. 1

D. 2

E. None of the above

13. (4 points) Find the area of the triangle $A(-1,-1,-1)$, $B(0,1,-1)$, $C(-1,1,2)$.

A. $\frac{5}{2}$

B. $\frac{7}{2}$

C. 3

D. 4

E. None of the above

14. (4 points) Find the parametrization of the curve of intersection of surface $x^2+y^2-z^2=0$ and plane $z-y=1$.

A. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in[0,2\pi]$

B. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in(-\infty,\infty)$

C. $x=t,\ y=\dfrac{t^2-1}{2},\ z=\dfrac{t^2+1}{2},\ t\in(-\infty,\infty)$

D. $x=t,\ y=\dfrac{t^2-1}{2},\ z=\dfrac{t^2+1}{2},\ t\in[0,2\pi]$

E. None of the above

15. (4 points) Find equation for the line in the plane $z=3$ that makes an angle of $\pi/6$ rad with $\mathbf{i}$ and an angle of $\pi/3$ rad with $\mathbf{j}$.

A. $x=\frac{\sqrt{3}}{2}t,\ \ y=\frac{1}{2}t,\ \ z=3$

B. $x=\frac{\sqrt{3}}{2}t,\ \ y=\frac{1}{2}t,\ \ z=-3$

C. $x=-\frac{1}{2}t,\ \ y=\frac{\sqrt{3}}{2}t,\ \ z=3$

D. $x=\frac{1}{2}t,\ \ y=\frac{\sqrt{3}}{2}t,\ \ z=-3$

E. None of the above

More Challenging Problem(s). Conceptual True/False. No work needed.

16. Are the following statements True or False in general? Circle your choice.

(a) (2 points) For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ and any scalar k, $k(\mathbf{u} \times \mathbf{v}) = (k\mathbf{u}) \times \mathbf{v}$.

A. TRUE

B. FALSE

(b) (3 points) For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $|\mathbf{u} + \mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$.

A. TRUE

B. FALSE

(c) (3 points) If $|\mathbf{r}(t)| = 1$ for all t then $|\mathbf{r}'(t)|$ is a constant.

A. TRUE

B. FALSE

(d) (3 points) If $\mathbf{r}(t)$ is a differentiable vector function, then $\frac{d}{dt}|\mathbf{r}(t)| = |\mathbf{r}'(t)|$.

A. TRUE

B. FALSE

(e) (3 points) If $f(x, y)$ is a function of x, y continuous at $(2, 5)$, then $\lim_{(x,y)\to(2,5)} f(x, y) = f(2, 5)$.

A. TRUE

B. FALSE

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET PAGE 1

Vectors in Space

Suppose $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$:

- **Unit Vectors**:
$$\mathbf{i} = \langle 1, 0, 0 \rangle$$
$$\mathbf{j} = \langle 0, 1, 0 \rangle$$
$$\mathbf{k} = \langle 0, 0, 1 \rangle$$
- **Length** of vector $\mathbf{u}$
$$|\mathbf{u}| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$
- **Dot Product**:
$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 = |\mathbf{u}||\mathbf{v}| \cos\theta$$
- **Cross Product**:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
- **Vector Projection**: $\text{proj}_{\mathbf{u}} \mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2} \mathbf{u}$

Partial Derivatives

- **Chain Rule**: Suppose $z = f(x, y)$ and $x = g(t)$ and $y = h(t)$ are all differentiable then
$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Curves and Planes in Space

- **Line** parallel to $\mathbf{v}$: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$
- **Plane** normal to $\mathbf{n} = \langle a, b, c \rangle$:
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
- **Arc Length** of curve $\mathbf{r}(t)$ for $t \in [a, b]$.
$$L = \int_a^b |\mathbf{r}'(t)| \, dt$$
- **Unit Tangent Vector** of curve $\mathbf{r}(t)$
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

Trigonometry

- $\sin^2 x + \cos^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$