Name: ____________________

Section: __________ **Recitation Instructor**: __________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

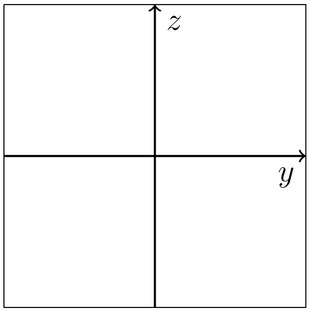
- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page for scratch work. Work on the back of pages will typically not be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

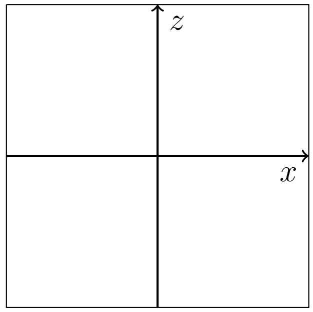
SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

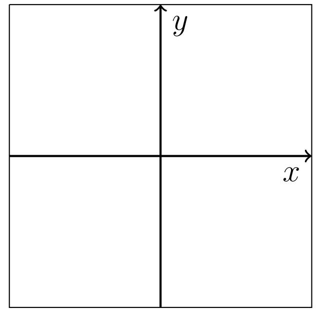
1. (7 points) Sketch the traces of the surface $-x^2 + y + z^2 = 1$ in coordinate planes and the surface itself. Determine the type of surface by circling one of the choices.



$yz-$plane

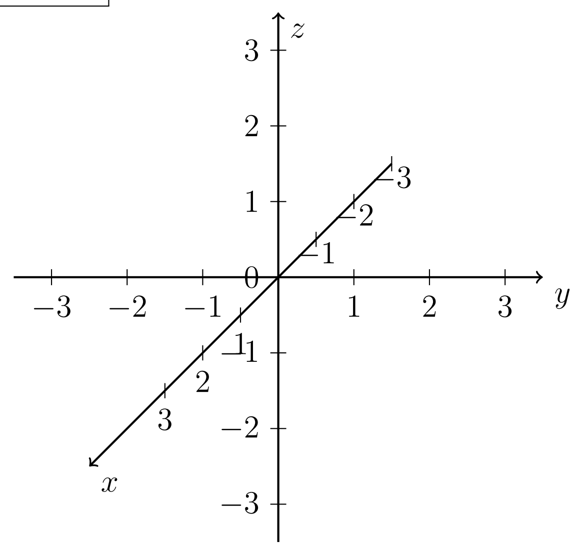


xz-plane



$xy-$plane

A. elliptic paraboloid
B. hyperbolic paraboloid
C. hyperboloid of one sheet
D. hyperboloid of two sheets
E. cone



2. (7 points) Evaluate $\displaystyle\lim_{\substack{(x,y)\to(2,0)\\ 2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{2x-y-4}$ or show that the limit does not exist .

3. (7 points) Find all the critical points of $f(x, y) = x^3 + 3xy + y^3$.
   Identify each critical point as a local maxima, a local minima or a saddle point.

4. (7 points) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4 + y^2}$ or show that the limit does not exist .

5. (14 points) Find the absolute maximum and minimum values of $f(x, y) = x + y - xy$ on the closed triangular region with vertices $(0, 0)$, $(0, 2)$ and $(4, 0)$.

6. Let $f(x, y) = \dfrac{x}{x+y}$ and answer the following questions

   (a) (5 points) Calculate the partial derivatives f_x and f_y.

   (b) (5 points) Use your answer in part (a) to find the linearization of f at $(2, 1)$.

   (c) (4 points) Use your answer in part (b) to approximate the number $f(2.2, 0.9)$.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

7. (4 points) Find the equation of the tangent plane to the surface:

$$x^2 + y^2 - z^2 = 18,$$

at the point $(3, 5, -4)$

A. $6x + 10y - 8z = 0$

B. $6x + 10y + 8z = 0$

C. $6(x - 3) + 10(y - 5) + 8(z + 4) = 0$

D. $3(x - 6) + 5(y - 5) - 4(z + 4) = 0$

E. None of the above

8. (4 points) Find the position $\mathbf{r}(t)$ of the particle in space at time $t \geq 0$ whose initial position is $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$ initial velocity $\mathbf{v}(0) = \langle 1, 0, 0 \rangle$, and acceleration is given by $\mathbf{a}(t) = \langle 2, 6t, -1 \rangle$

A. $\langle t + t^2, 1 + t^3, t^2 \rangle$

B. $\langle t + t^2, 1 + t^3, -\frac{t^2}{2} \rangle$

C. $\langle t + t^2, t + t^3, \frac{t^2}{2} \rangle$

D. $\langle 1 + t + t^2, 1 + t^3, -\frac{t^2}{2} \rangle$

E. None of the above

9. (4 points) A particle's motion is described by the parametric equations $x(t) = e^t \cos t,\ y(t) = e^t \sin t$, $z(t) = e^t$. How far does the particle travel during $t \in [0, 1]$?

A. $\sqrt{3} \cdot (e - 1)$

B. $\sqrt{3} \cdot e$

C. $\sqrt{3} \cdot (e + 1)$

D. $\sqrt{3} \cdot (e + 2)$

E. None of the above

10. (4 points) Find the value of $\partial z/\partial x$ at the point $(1,1,1)$ if z as a function of x and y is defined by the equation $z^3 - xy + yz + y^3 - 2 = 0$.

    A. $\frac{1}{4}$

    B. -1

    C. $-\frac{1}{4}$

    D. 1

    E. None of the above

11. (4 points) Let $z = f(x,y)$, where f is differentiable, and $x = g(t)$ and $y = h(t)$. Suppose that $g(3) = 2$, $h(3) = 7$, $g'(3) = 5$, $h'(3) = -4$, $f_x(2,7) = 6$ and $f_y(2,7) = -8$. Find $\frac{dz}{dt}$ when $t = 3$.

    A. -64

    B. -44

    C. 62

    D. 64

    E. None of the above

12. (4 points) Consider the function $f(x,y) = x^2 + xy + y^2$ at the point $(-1,1)$. In what direction does f decrease most rapidly?

    A. $\frac{1}{\sqrt{2}}\langle 1, 1\rangle$

    B. $\frac{1}{\sqrt{2}}\langle 1, -1\rangle$

    C. $\frac{1}{\sqrt{2}}\langle -1, 1\rangle$

    D. $\frac{1}{\sqrt{2}}\langle -1, -1\rangle$

    E. $\langle 0, 1\rangle$

13. (4 points) Find the distance between two planes given by equations $x+2y-z=1$ and $-x-2y+z=-7$.

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. $\sqrt{6}$

E. None of the above

14. (4 points) Find the parametrization of the curve of intersection of surface $z=x^2+y^2$ and plane $x+y=1$.

A. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in[0,2\pi]$

B. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in(-\infty,\infty)$

C. $x=t,\ y=1-t,\ z=2t^2-2t+1,\ t\in(-\infty,\infty)$

D. $x=t,\ y=1+t,\ z=t^2+1,\ t\in(-\infty,\infty)$

E. None of the above

15. (4 points) Find equation for the line orthogonal to two lines $x=1+t,\ y=2,\ z=2-t$ and $x=1,\ y=2+t,\ z=2+t$ passing through the point $P(1,2,2)$.

A. $x=1+t,\ y=2-t,\ z=2+t$

B. $x=1+2t,\ y=2-t,\ z=2+t$

C. $x=1-t,\ y=2-t,\ z=2+t$

D. $x=t,\ y=2,\ z=-1+t$

E. None of the above

More Challenging Question(s). Show all work to receive credit.

16. Let $f(x, y) = x^2 + 4y^2 - 4xy + 2$

(a) (5 points) Show that $f(x, y)$ has an infinite number of critical points.

(b) (3 points) Show that the second derivative test is inconclusive at each critical point.

(c) (6 points) Then show that f has a local and absolute minimum at each critical point.

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET PAGE 1

Vectors in Space

Suppose $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$:

- **Unit Vectors**:
$$\mathbf{i} = \langle 1, 0, 0 \rangle$$
$$\mathbf{j} = \langle 0, 1, 0 \rangle$$
$$\mathbf{k} = \langle 0, 0, 1 \rangle$$
- **Length** of vector $\mathbf{u}$
$$|\mathbf{u}| = \sqrt{{u_1}^2 + {u_2}^2 + {u_3}^2}$$
- **Dot Product**:
$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= u_1 v_1 + u_2 v_2 + u_3 v_3 \\ &= |\mathbf{u}||\mathbf{v}| \cos\theta \end{aligned}$$
- **Cross Product**:
$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$
- **Vector Projection**: $\text{proj}_{\mathbf{u}}\mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|^2}\, \mathbf{u}$

Partial Derivatives

- **Chain Rule**: Suppose $z = f(x, y)$ and $x = g(t)$ and $y = h(t)$ are all differentiable then
$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Curves and Planes in Space

- **Line** parallel to $\mathbf{v}$: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$
- **Plane** normal to $\mathbf{n} = \langle a, b, c \rangle$:
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
- **Arc Length** of curve $\mathbf{r}(t)$ for $t \in [a, b]$.
$$L = \int_a^b |\mathbf{r}'(t)|\, dt$$
- **Unit Tangent Vector** of curve $\mathbf{r}(t)$
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

Surfaces

- **Directional Derivative:** $D_{\mathbf{u}} f(x, y) = \nabla f \cdot \mathbf{u}$
- **Second Derivative Test** Suppose $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let
$$D = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$
  - (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
  - (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
  - (c) If $D < 0$ then $f(a, b)$ is a saddle point.

Trigonometry

- $\sin^2 x + \cos^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$