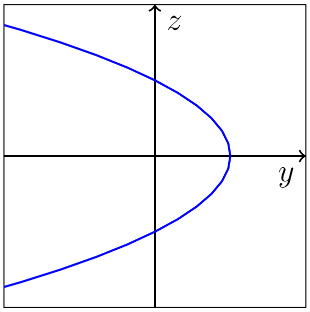
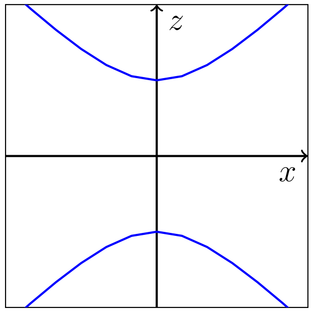
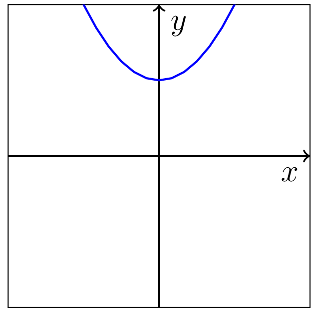
Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. (7 points) Sketch the traces of the surface $-x^2 + y + z^2 = 1$ in coordinate planes and the surface itself. Determine the type of surface by circling one of the choices.







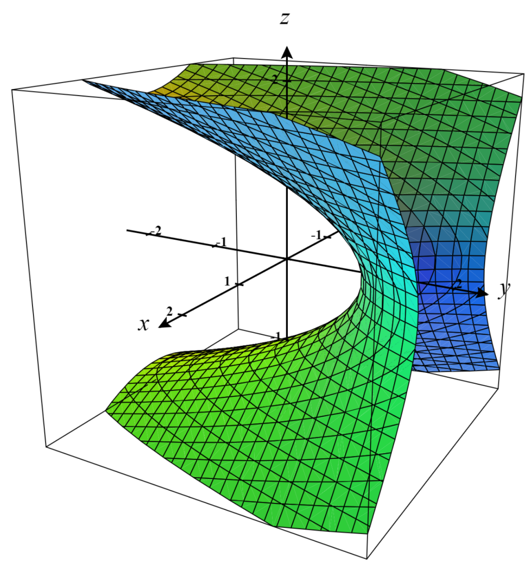
A. elliptic paraboloid

B. hyperbolic paraboloid

C. hyperboloid of one sheet

D. hyperboloid of two sheets

E. cone



2. (7 points) Evaluate $\displaystyle\lim_{\substack{(x,y)\to(2,0)\\2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{2x-y-4}$ or show that the limit does not exist .

Solution:

$$\lim_{\substack{(x,y)\to(2,0)\\2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{2x-y-4}\cdot\left(\frac{\sqrt{2x-y}+2}{\sqrt{2x-y}+2}\right) = \lim_{\substack{(x,y)\to(2,0)\\2x-y\neq 4}} \frac{2x-y-4}{2x-y-4}\cdot\left(\frac{1}{\sqrt{2x-y}+2}\right)$$

$$= \lim_{\substack{(x,y)\to(2,0)\\2x-y\neq 4}} \frac{1}{\sqrt{2x-y}+2} = \boxed{\frac{1}{4}}$$

3. (7 points) Find all the critical points of $f(x, y) = x^3 + 3xy + y^3$.
Identify each critical point as a local maxima, a local minima or a saddle point.

Solution:

$$f_x = 3x^2 + 3y \qquad f_y = 3x + 3y^2$$

$$f_{xx} = 6x \qquad f_{xy} = 3 \qquad f_{yy} = 6y$$

So since f_x and f_y are never undefined the only critical points of f are when both $3x^2 + 3y = 0$ and $3x + 3y^2 = 0$. This occurs at $(0, 0)$ and $(-1, -1)$.

$$D(0, 0) = (0)(0) - (3)^2 = -9 < 0$$

so $(0, 0)$ is a saddle point.

$$D(-1, -1) = (-6)(-6) - (3)^2 = 27 > 0$$
$$f_{xx}(-1, -1) - -6 < 0$$

so $(-1, -1)$ is a local max.

4. (7 points) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4 + y^2}$ or show that the limit does not exist .

Solution: We will use the two path test to show that the limit does not exist.

Path 1: $y = 0, x \to 0$

$$\lim_{y=0,x\to 0} \frac{x^4}{x^4 + y^2} = \lim_{x\to 0} \frac{x^4}{x^4} = 1$$

Path 2: $x = 0, y \to 0$

$$\lim_{x=0,y\to 0} \frac{x^4}{x^4 + y^2} = \lim_{y\to 0} \frac{0}{y^2} = 0$$

So by the two path test $\boxed{\text{the limit does not exist}}$

5. (14 points) Find the absolute maximum and minimum values of $f(x, y) = x + y - xy$ on the closed triangular region with vertices $(0, 0)$, $(0, 2)$ and $(4, 0)$.

Solution:

$$f_x = 1 - y \qquad\qquad f_y = 1 - x$$

So the only critical point on the interior of the region is $(1, 1)$. Now for along the border:

Bottom Side: $y = 0, x \in [0, 4]$

$f(x, 0) = x$. No critical points. Endpoints $(0, 0)$ and $(4, 0)$ should be included in list of possible absolute min/max.

Left Side: $x = 0, y \in [0, 2]$

$f(0, y) = y$. No critical points. Endpoint $(0, 2)$ should be included in list of possible absolute min/max.

Hypotenuse: $y = 2 - \frac{1}{2}x, x \in [0, 4]$

$f(x, 2 - \frac{1}{2}x) = x + (2 - \frac{1}{2}x) - x(2 - \frac{1}{2}x)$.
$f_x(x, 2 - \frac{1}{2}x) = -\frac{3}{2} + x$
So there is a critical point at $(\frac{3}{2}, \frac{5}{4})$ that should be included in list of possible absolute min/max.

Finally we check the function values for each possible absolute min/max

$$\begin{aligned} &f(1, 1) = 1 \\ &f(0, 0) = 0 \leftarrow \text{absolute min} \\ &f(4, 0) = 4 \leftarrow \text{absolute max} \\ &f(0, 2) = 2 \\ &f(\tfrac{3}{2}, \tfrac{5}{4}) = 7/8 \end{aligned}$$

6. Let $f(x,y) = \dfrac{x}{x+y}$ and answer the following questions

(a) (5 points) Calculate the partial derivatives f_x and f_y.

Solution:

$$f_x = \frac{1(x+y) - x(1)}{(x+y)^2} = \frac{y}{(x+y)^2}$$
$$f_y = \frac{0(x+y) - x(1)}{(x+y)^2} = \frac{-x}{(x+y)^2}$$

(b) (5 points) Use your answer in part (a) to find the linearization of f at $(2,1)$.

Solution:

$$f(2,1) = \frac{2}{3} \qquad f_x(2,1) = \frac{1}{9} \qquad f_y(2,1) = \frac{-2}{9}$$

So therefore $\boxed{L(x,y) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1)}$

(c) (4 points) Use your answer in part (b) to approximate the number $f(2.2, 0.9)$.

Solution:

$$\begin{aligned} L(2.2, 0.9) &= \tfrac{2}{3} + \tfrac{1}{9}(2.2-2) - \tfrac{2}{9}(0.9-1) \\ &= \tfrac{2}{3} + \tfrac{1}{9}(\tfrac{2}{10}) - \tfrac{2}{9}(\tfrac{-1}{10}) \\ &= \boxed{\tfrac{2}{3} + \tfrac{4}{90}} \end{aligned}$$

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

7. (4 points) Find the equation of the tangent plane to the surface:

$$x^2 + y^2 - z^2 = 18,$$

at the point $(3, 5, -4)$

A. $6x + 10y - 8z = 0$

B. $6x + 10y + 8z = 0$

C. $6(x-3) + 10(y-5) + 8(z+4) = 0$

D. $3(x-6) + 5(y-5) - 4(z+4) = 0$

E. None of the above

8. (4 points) Find the position $\mathbf{r}(t)$ of the particle in space at time $t \geq 0$ whose initial position is $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$ initial velocity $\mathbf{v}(0) = \langle 1, 0, 0 \rangle$, and acceleration is given by $\mathbf{a}(t) = \langle 2, 6t, -1 \rangle$

A. $\langle t + t^2, 1 + t^3, t^2 \rangle$

B. $\langle t + t^2, 1 + t^3, -\frac{t^2}{2} \rangle$

C. $\langle t + t^2, t + t^3, \frac{t^2}{2} \rangle$

D. $\langle 1 + t + t^2, 1 + t^3, -\frac{t^2}{2} \rangle$

E. None of the above

9. (4 points) A particle's motion is described by the parametric equations $x(t) = e^t \cos t$, $y(t) = e^t \sin t$, $z(t) = e^t$. How far does the particle travel during $t \in [0, 1]$?

A. $\sqrt{3} \cdot (e - 1)$

B. $\sqrt{3} \cdot e$

C. $\sqrt{3} \cdot (e + 1)$

D. $\sqrt{3} \cdot (e + 2)$

E. None of the above

10. (4 points) Find the value of $\partial z/\partial x$ at the point $(1,1,1)$ if z as a function of x and y is defined by the equation $z^3 - xy + yz + y^3 - 2 = 0$.

 A. $\frac{1}{4}$

 B. -1

 C. $-\frac{1}{4}$

 D. 1

 E. None of the above

11. (4 points) Let $z = f(x,y)$, where f is differentiable, and $x = g(t)$ and $y = h(t)$. Suppose that $g(3) = 2$, $h(3) = 7$, $g'(3) = 5$, $h'(3) = -4$, $f_x(2,7) = 6$ and $f_y(2,7) = -8$. Find $\frac{dz}{dt}$ when $t = 3$.

 A. -64

 B. -44

 C. 62

 D. 64

 E. None of the above

12. (4 points) Consider the function $f(x,y) = x^2 + xy + y^2$ at the point $(-1,1)$. In what direction does f decrease most rapidly?

 A. $\frac{1}{\sqrt{2}}\langle 1,1\rangle$

 B. $\frac{1}{\sqrt{2}}\langle 1,-1\rangle$

 C. $\frac{1}{\sqrt{2}}\langle -1,1\rangle$

 D. $\frac{1}{\sqrt{2}}\langle -1,-1\rangle$

 E. $\langle 0,1\rangle$

13. (4 points) Find the distance between two planes given by equations $x+2y-z=1$ and $-x-2y+z=-7$.

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. $\sqrt{6}$

E. None of the above

14. (4 points) Find the parametrization of the curve of intersection of surface $z=x^2+y^2$ and plane $x+y=1$.

A. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in[0,2\pi]$

B. $x=\cos t,\ y=\sin t,\ z=1+\sin t,\ t\in(-\infty,\infty)$

C. $x=t,\ y=1-t,\ z=2t^2-2t+1,\ t\in(-\infty,\infty)$

D. $x=t,\ y=1+t,\ z=t^2+1,\ t\in(-\infty,\infty)$

E. None of the above

15. (4 points) Find equation for the line orthogonal to two lines $x=1+t,\ y=2,\ z=2-t$ and $x=1,\ y=2+t,\ z=2+t$ passing through the point $P(1,2,2)$.

A. $x=1+t,\ y=2-t,\ z=2+t$

B. $x=1+2t,\ y=2-t,\ z=2+t$

C. $x=1-t,\ y=2-t,\ z=2+t$

D. $x=t,\ y=2,\ z=-1+t$

E. None of the above

More Challenging Question(s). Show all work to receive credit.

16. Let $f(x, y) = x^2 + 4y^2 - 4xy + 2$

(a) (5 points) Show that $f(x, y)$ has an infinite number of critical points.

Solution:

$$f_x = 2x - 4y \qquad f_y = 8y - 4x$$

So $f_x = 0$ when $2y = x$ and $f_y = 0$ when $2y = x$. So anywhere along the line $2y = x$ is a critical point

(b) (3 points) Show that the second derivative test is inconclusive at each critical point.

Solution:

$$f_{xx} = 2 \qquad f_{xy} = -4 \qquad f_{yy} = 8$$

So $D(xy) = 2(8) - (-4)^2 = 0$ so the second derivative test is inconclusive.

(c) (6 points) Then show that f has a local and absolute minimum at each critical point.

Solution: We can rewrite f as $f(x, y) = (x - 2y)^2 + 2$. From this form we can see that the lowest possible output for f is 2 and it occurs at anytime $x = 2y$. Therefore each points on the line is an absolute (and local) minimum.