Name: ______________________ **PID:** ______________

Section: ____________ **Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 13.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- You will be given exactly 120 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________

SIGNATURE

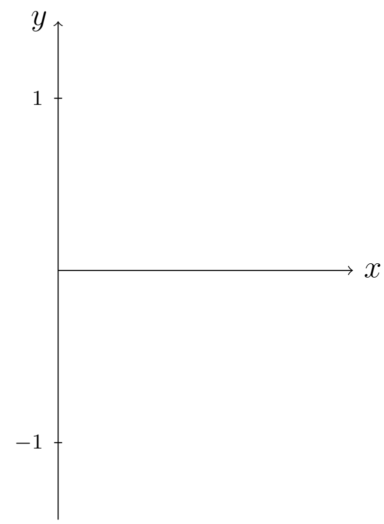
Page	2	3	4	5	6	7	8	9	10	11	Total
Points	/12	/12	/12	/12	/12	/9	/9	/9	/9	/12	/100

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Let R be the region in the first quadrant bounded by $y = \sin x$, $y = \cos x$, and $x = 0$.

   (a) (2 points) Sketch the region R on the axes to the right.

   (b) (3 points) Set up BUT DO NOT EVALUATE the definite integral representing the volume of the resulting solid if the region R is rotated about the x-axis.



2. (7 points) Evaluate the integral $\displaystyle\int_0^3 \frac{dx}{9 + x^2}$

3. Evaluate the following integrals.

(a) (6 points) $\int \sec^3 x \tan x \; dx$

(b) (6 points) $\int y \cdot \ln y \; dy$

4. Determine whether each of the series below is convergent or divergent. For full credit, you must show your work and indicate which test(s) you used.

   (a) (6 points) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln(3n)}$

   (b) (6 points) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n\sqrt{n+1}}$

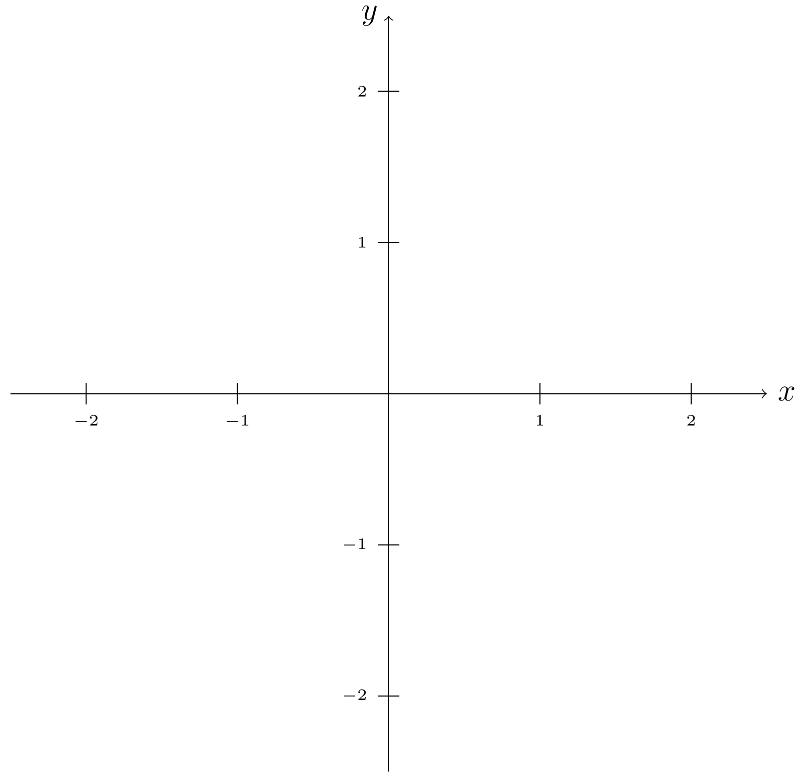
5. (6 points) Evaluate $\int \frac{\sin x}{x}\,dx$ as an infinite series. Express your answer in sigma notation.

6. (6 points) Consider the curve C given by the parametric equations:

$$x = 2 + 3t, \quad y = \cosh(3t)$$

Find the arc length of the curve on the interval $0 \le t \le 1$.

7. Two curves in polar coordinates are given by equations $r = 1$ and $r = 1 - \sin\theta$, respectively.



(a) (3 points) Sketch both curves.

(b) (3 points) Determine the intersection points of these two curves. Express these points in polar coordinates (r, θ).

(c) (6 points) Set up BUT DO NOT EVALUATE the definite integral representing the area that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \sin\theta$.

Multiple Choice. Circle the best answer. No work needed.
No partial credit available. No credit will be given for choices not clearly marked.

8. (3 points) Suppose $\sum_{n=3}^{\infty} a_n$ is a convergent series with positive terms.

   Which of the following does not necessarily converge?

   A. $\sum_{n=3}^{\infty} 500 \cdot a_n$

   B. $\sum_{n=3}^{\infty} \frac{a_n}{\ln n}$

   C. $\sum_{n=3}^{\infty} a_n \cdot (-1)^n$

   D. $\sum_{n=3}^{\infty} (a_n)^2$

   E. $\sum_{n=3}^{\infty} \sqrt{a_n}$

9. (3 points) Determine the sum of the series $1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!} + \cdots$.

   A. 1

   B. e^{-1}

   C. 0

   D. e

   E. ∞

10. (3 points) A mountain climber is about to haul up a 100 m length of a hanging rope. How much work will it take if the rope weighs 1 N/m?

    A. 500 N·m

    B. 1000 N·m

    C. 5000 N·m

    D. 10000 N·m

    E. 50000 N·m

11. (3 points) Evaluate the integral $\int \frac{x}{1+x^2}\,dx$

A. $\frac{x^2}{2}\tan^{-1}(x) + C$
B. $x\tan^{-1}(x) + C$
C. $x\ln(1+x^2) + C$
D. $\frac{1}{2}\ln(1+x^2) + C$
E. None of the above

12. (3 points) Find $f'(x)$ for the function $f(x) = x^x$ $(x > 0)$.

A. $f'(x) = x^x(1 + \ln x)$
B. $f'(x) = x^x(x + 1)$
C. $f'(x) = x \cdot x^{x-1}$
D. $f'(x) = x^x \cdot \ln x$
E. None of the above

13. (3 points) Find $f'(x)$ for the function $f(x) = \cosh(\ln x)$

A. $f'(x) = \sinh(\ln x)$
B. $f'(x) = \frac{1}{2} - \frac{1}{2x^2}$
C. $f'(x) = \frac{\sinh(\ln x)}{x}$
D. $f'(x) = \frac{1}{2} + \frac{1}{2x^2}$
E. $f'(x) = \cosh(\ln x)$

14. (3 points) Express $\dfrac{1}{(x^2+1)(x^2-1)}$ as a sum of partial fractions.

A. $\dfrac{Ax}{x^2+1}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$

B. $\dfrac{A}{x^2+1}+\dfrac{B}{x-1}$

C. $\dfrac{Ax}{x+1}+\dfrac{B}{x^2+1}+\dfrac{C}{x-1}$

D. $\dfrac{Ax+B}{x^2+1}+\dfrac{Cx}{x^2-1}$

E. $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-1}+\dfrac{D}{x+1}$

15. (3 points) Evaluate the integral $\displaystyle\int_1^2 \frac{dx}{\sqrt{x-1}}$.

A. 2

B. 1

C. 0

D. ∞

E. Cannot be determined

16. (3 points) Evaluate $\displaystyle\lim_{x\to 0^+} x^{\sqrt{x}}$.

A. 0

B. ∞

C. e

D. e^{-1}

E. 1

17. (3 points) Determine $y(e)$ if $\dfrac{dy}{dx} = \dfrac{\ln x}{xy}$ and $y(1) = 2$.

   A. 5
   B. $-\sqrt{3}$
   C. $\sqrt{5}$
   D. $\sqrt{3}$
   E. $-\sqrt{5}$

18. (3 points) Determine the open interval of convergence of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(k+1)x^{2k-1}}{3^k}$.

   A. $(0, 3)$
   B. $(-\infty, \infty)$
   C. $(-9, 9)$
   D. $(-3, 3)$
   E. $(-\sqrt{3}, \sqrt{3})$

19. (3 points) Consider the series $\displaystyle\sum_{n=1}^{\infty} |\sin n|$. Which of the following statements is true?

   A. The integral test says that the series converges.
   B. The integral test says that the series diverges.
   C. The integral test does not apply for this series as the terms are not decreasing.
   D. The integral test does not apply for this series as the terms are not always positive.
   E. The integral test does not apply for this series as the function $f(x) = |\sin x|$ is not continuous.

Conceptual True/False Question(s). Circle only the correct answer.

20. (2 points) $\displaystyle\int_1^\infty \frac{dx}{x^{\sqrt{2}}}$ is convergent

    A. TRUE

    B. FALSE

21. (2 points) $\cosh x \geq 1$ for all x

    A. TRUE

    B. FALSE

22. (2 points) The equations in polar coordinates $r = 1 - \sin(2\theta)$ and $r = \sin(2\theta) - 1$ describe the same curve.

    A. TRUE

    B. FALSE

23. (3 points) The parametric equations $x = t^2$, $y = t^4$ and $x = t^3$, $y = t^6$ describe the same curve.

    A. TRUE

    B. FALSE

24. (3 points) If the sequences $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent

    A. TRUE

    B. FALSE

FORMULA SHEET PAGE 1

Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by
$$V = \int_a^b A(x)\ dx$$
- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:
$$W = \int_a^b f(x)\ dx$$
- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$
- $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\displaystyle\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad \text{for } a \neq 1$
- **Integration by Parts**:
$$\int u\ dv = uv - \int v\ du$$
- Arc Length Formula:
$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$

Derivatives

- $\dfrac{d}{dx}(\sinh x) = \cosh x \qquad \dfrac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$
- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and
$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions
$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$
$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$
- $\cosh^2 x - \sinh^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2 \sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Parametric

- $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \text{if} \quad \dfrac{dx}{dt} \neq 0$
- Arc Length: $\displaystyle L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$

Polar

- $x = r\cos\theta \qquad y = r\sin\theta$
- $r^2 = x^2 + y^2 \qquad \tan\theta = \dfrac{y}{x}$
- Area: $\displaystyle A = \int_a^b \frac{1}{2} r(\theta)^2\ d\theta$
- $\displaystyle L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$

FORMULA SHEET PAGE 2

Series

- **nth term test for divergence:** If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

- **The p-series:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

- **Geometric:** If $|r| < 1$ then $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

- **The Integral Test:** Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then
  - (i) If $\int_1^{\infty} f(x)\, dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
  - (ii) If $\int_1^{\infty} f(x)\, dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

- **The Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
  - (i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
  - (ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

- **The Limit Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If
$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$
where c is a finite number and $c > 0$, then either both series converge or both diverge.

- **Alternating Series Test:** If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ satisfies
  - (i) $0 < b_{n+1} \leq b_n$ for all n
  - (ii) $\lim_{n\to\infty} b_n = 0$

  then the series is convergent.

- **The Ratio Test**
  - (i) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
  - (ii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
  - (iii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive.

- **Maclaurin Series**: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}\, x^n$

- **Taylor's Inequality** If $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality
$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{for } |x-a| \leq d$$

- Some Power Series
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\qquad R = \infty$
  - $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\qquad R = \infty$
  - $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\qquad R = \infty$
  - $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ $\qquad R = 1$
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $\qquad R = 1$