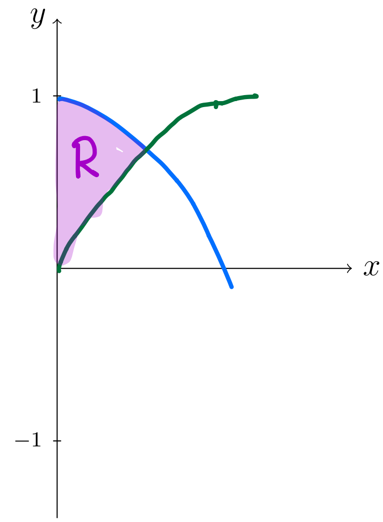
Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. Let R be the region in the first quadrant bounded by $y = \sin x$, $y = \cos x$, and $x = 0$.

   (a) (2 points) Sketch the region R on the axes to the right.

   (b) (3 points) Set up BUT DO NOT EVALUATE the definite integral representing the volume of the resulting solid if the region R is rotated about the x-axis.



$$\pi \int_0^{\pi/4} \cos^2 x - \sin^2 x \, dx$$

2. (7 points) Evaluate the integral $\displaystyle\int_0^3 \frac{dx}{9+x^2}$

$$\frac{1}{9}\int_0^3 \frac{1}{1+\left(\frac{x}{3}\right)^2} dx$$

$u = x/3 \quad du = dx/3$

$$\frac{1}{3}\int \frac{1}{1+u^2} du$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big|_0^3$$

$$= \frac{1}{3}\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$= \frac{1}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{12}$$

$x = 3\tan\theta \qquad dx = 3\sec^2\theta \, d\theta$

$$\int \frac{3\sec^2\theta \, d\theta}{9 + 9\tan^2\theta}$$

$$\int \frac{3\cancel{\sec^2\theta}\, d\theta}{9\cancel{\sec^2\theta}} = \frac{1}{3}\theta$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

So

$$= \frac{1}{3}\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$= \frac{1}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{12}$$

3. Evaluate the following integrals.

(a) (6 points) $\int \sec^3 x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x$

$\int u^2 \, du = \frac{1}{3} u^3 + C$

$= \frac{1}{3} \sec^3 x + C$

(b) (6 points) $\int y \cdot \ln y \, dy$

$u = \ln y \qquad dv = y \, dy$

$du = \frac{1}{y} \, dy \qquad v = \frac{1}{2} y^2$

$= \frac{1}{2} y^2 \cdot \ln y - \int \frac{1}{2} y^{\cancel{2}} \cdot \cancel{\frac{1}{y}} \, dy$

$= \frac{1}{2} y^2 \cdot \ln y - \frac{1}{2} \left(\frac{1}{2} y^2 \right) + C$

$= \frac{1}{2} y^2 \cdot \ln y - \frac{1}{4} y^2 + C$

4. Determine whether each of the series below is convergent or divergent. For full credit, you must show your work and indicate which test(s) you used.

(a) (6 points) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln(3n)}$ (for $n \geq 2$)

$\ln(3n) \leq 3n \Rightarrow \frac{1}{3n} \leq \frac{1}{\ln(3n)} \Rightarrow \frac{\sqrt{n}}{3n} \leq \frac{\sqrt{n}}{\ln(3n)}$

so since $\sum \frac{\sqrt{n}}{3n} = \frac{1}{3} \sum \frac{1}{n^{1/2}}$ diverges by p-series

we know $\sum \frac{\sqrt{n}}{\ln(3n)}$ diverges by DCT.

(b) (6 points) $\sum_{n=2}^{\infty} \frac{2}{n\sqrt{n+1}}$ Use LCT with $b_n = \frac{1}{n\sqrt{n}} = \frac{1}{n^{3/2}}$

$\lim_{n\to\infty} \left[\frac{2}{n\sqrt{n+1}} \cdot \frac{n\sqrt{n}}{1} \right] = \lim_{n\to\infty} 2 \cdot \frac{n}{n} \cdot \sqrt{\frac{n}{n+1}}$

$= 2 \cdot 1 \cdot \sqrt{1} = 2$

So since $\sum \frac{1}{n^{3/2}}$ conv by p-series,

$\sum \frac{2}{n\sqrt{n+1}}$ must also conv by LCT.

5. (6 points) Evaluate $\int \frac{\sin x}{x}\, dx$ as an infinite series. Express your answer in sigma notation.

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\frac{\sin x}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$$

$$\int \frac{\sin x}{x} = \sum_{n=0}^{\infty} \left[\frac{(-1)}{(2n+1)!} \cdot \frac{x^{2n+1}}{(2n+1)} \right] + C$$

6. (6 points) Consider the curve C given by the parametric equations:

$$x = 2 + 3t, \quad y = \cosh(3t)$$

Find the arc length of the curve on the interval $0 \leq t \leq 1$.

$$x' = 3 \qquad y' = \sinh(3t) \cdot 3$$

$$L = \int_0^1 \sqrt{9 + 9\sinh^2(3t)}\, dt$$

$$= \int_0^1 \sqrt{9\cosh^2(3t)}\, dt$$

$$= \int_0^1 3\cosh(3t)\, dt = \sinh(3t)\Big|_0^1$$

$$= \sinh(3)$$

7. Two curves in polar coordinates are given by equations $r = 1$ and $r = 1 - \sin\theta$, respectively.
   (a) (3 points) Sketch both curves.

   Solution: See here.

   (b) (3 points) Determine the intersection points of these two curves. Express these points in polar coordinates (r, θ).

   Solution: The points of intersection occur at $(1, 0)$ and $(1, \pi)$.

   (c) (6 points) Set up BUT DO NOT EVALUATE the definite integral representing the area that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \sin\theta$.

   Solution:

$$\int_0^{\pi/2} (1 - (1 - \sin\theta)^2)\, d\theta$$

8. E
9. B
10. C
11. D
12. A
13. B, C
14. E
15. A
16. E
17. C
18. E
19. C
20. A
21. A
22. A
23. B
24. B