Fall 2014 **MTH 234 FINAL EXAM** December 8, 2014

Name: ______________________________ **PID:** ______________________

Section: ______________ **Instructor**: ______________________________

DO NOT WRITE BELOW THIS LINE. Go to the next page.

Page	Problem	Score	Max Score
1	1		5
	2		5
	3		5
	4		5
2	5		5
	6		5
	7		5
	8		5
	9		5
	10		5
3	11		10
	12a		8
	12b		4
4	13		16
5	14a		4
	14b		10
	14c		6
6	15a		8
	15b		8
7	16a		6
	16b		6
	17		12
8	18		12
	19		12
9	20		12
	21		16
Total Score			200

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READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything excepts pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 10.
- Fill in your name, etc. on the first page and on this page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- You will be given exactly 120 minutes for this exam.

I have read and understand the above instructions: ______________________

SIGNATURE

SCORE: ☐

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

1. A parametric equation for the line through $(1,2,3)$ and perpendicular to the plane $z-x-y=5$ is given by:

   (a) $\mathbf{r}(t)=\langle t-1, 2t-1, 3t+1\rangle$

   (b) $\mathbf{r}(t)=\langle -1-t, -2-t, -t+3\rangle$

   (c) $\mathbf{r}(t)=\langle -t-1, -2t-1, -3t+1\rangle$

   (d) $\mathbf{r}(t)=\langle 1-t, 2-t, 3+t\rangle$

   (e) None of the above

2. Suppose $x^2+3xy=7$. Which of the following is true?

   (a) $\dfrac{dy}{dx}=\dfrac{3x}{2x+3y}$

   (b) $\dfrac{dy}{dx}=\dfrac{2x+3y}{3x}$

   (c) $\dfrac{dy}{dx}=-\dfrac{3y}{2x+3y}$

   (d) $\dfrac{dy}{dx}=-\dfrac{2x+3y}{3x}$

   (e) None of the above

3. Let $f=x^2+3xy$ where $x=-v+\sin u$ and $y=u+\sin v$. Which of the following is true?

   (a) $\dfrac{\partial f}{\partial u}=(3u-2v+5\sin v)(\cos u)+(-3v+3\sin u)$

   (b) $\dfrac{\partial f}{\partial u}=(3u-2v+2\sin u+3\sin v)(\cos u)+(-3v+3\sin u)$

   (c) $\dfrac{\partial f}{\partial u}=(-3u+2v-2\sin u-3\sin v)+(-3v+3\sin u)(\cos v)$

   (d) $\dfrac{\partial f}{\partial u}=2(-v+\sin u)+3(u+\sin v)$

   (e) None of the above

4. The directional derivative of $f(x,y)=x^2(y-1)-2y^2$ at the point $(2,0)$ in the direction of the vector $\mathbf{i}-2\mathbf{j}$ is given by:

   (a) $\big\langle 2x(y-1), x^2-4y\big\rangle\cdot\dfrac{\langle 1,-2\rangle}{\sqrt{5}}$

   (b) $\big\langle 2x(y-1), x^2-4y\big\rangle\cdot\dfrac{\langle 2,0\rangle}{\sqrt{4}}$

   (c) $\langle -4,4\rangle\cdot\dfrac{\langle 1,-2\rangle}{\sqrt{5}}$

   (d) $\langle -6,9\rangle\cdot\dfrac{\langle 2,0\rangle}{\sqrt{4}}$

   (e) None of the above

Fill in the Blanks. No work needed. Only possible scores given are 0, 3, and 5.

5. Evaluate the limit if it exists. Write DNE if the limit does not exist. $\lim_{(x,y)\to(1,2)} \dfrac{4x^2 - y^2}{2x - y} =$ ______

6. Evaluate the limit if it exists. Write DNE if the limit does not exist. $\lim_{(x,y)\to(0,0)} \dfrac{4x^2 + y^2}{2x^2 - y^2} =$ ______

7. If $\dfrac{d\mathbf{r}}{dt} = (3t^2 + 1)\mathbf{i} + (4t^3 + 1)\mathbf{j} - \mathbf{k}$ and $\mathbf{r}(1) = -3\mathbf{i} + 2\mathbf{k}$ then $\mathbf{r}(t) =$ ______________________

8. The integral ______________________ expresses the length of the curve $\mathbf{r}(t) = \left\langle 1 + t^2, e^t, \sin(\pi t) \right\rangle$ between the points $(1, 1, 0)$ and $(2, e, 0)$. (**Do not evaluate**)

9. ______________________ is an equation of the tangent plane to the surface $z = x^2(y - 1) - 2y^2$ at the point $(2, 0, -4)$.

10. If $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^3 f(x, y, z)\ dz\ dy\ dx = \int_0^3 \int_0^a \int_0^b f(x, y, z)\ dx\ dy\ dz$ then $a =$________ and $b =$________.

Extra Work Space.

Standard Response Questions. Show all work to receive credit. Please put your final answer in the $\boxed{\textbf{BOX}}$.

11. (10 points) Find where the lines $L_1(t) = \langle 4+t, 5+t, 6+t \rangle$ and $L_2(s) = \langle 3-s, 6-2s, 1+s \rangle$ intersect

12. (8+4=12 points) Given the points $A(1,1,1)$, $B(2,1,0)$, and $C(0,2,3)$.

    (a) Find an equation of a plane that through the points A, B, and C.

    (b) Find the area of triangle ABC.

13. (16 points) Find and classify (max, min or saddle point) the critical points of $f(x, y) = 1 + 6x^2 + 6y^2 - 3x^2y - y^3$. (Hint: There are four.)

14. (4+10+6=20 points) Let $\mathbf{F} = \langle 2x + y + yz, x + xz, xy + 1 \rangle$.

(a) Show that $\text{curl}(\mathbf{F}) = \mathbf{0}$.

(b) Find a potential function f such that $\nabla f = \mathbf{F}$.

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a curve from the point $(0, 0, 0)$ to the point $(2, 1, 1)$.

15. (8+8=16 points) Evaluate the integrals

(a) $\displaystyle\int_0^1 \int_x^1 4e^{(x/y)}\, dy\, dx.$

(b) $\displaystyle\int_0^3 \int_0^{\sqrt{9-x^2}} 2\cos(x^2+y^2)\ dy\ dx.$

16. (6+6=12 points)

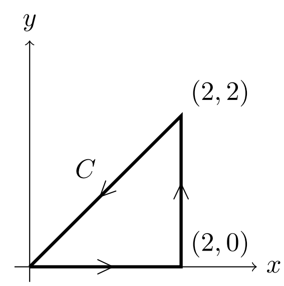
(a) Express $(x-2)^2 + y^2 = 4$ in terms of polar coordinates. Solve for r. Simplify as much as possible.

(b) Express (DO NOT EVALUATE) a triple integral in cylindrical coordinates for the volume of the portion of the sphere $x^2 + y^2 + z^2 = 16$ contained within the cylinder $(x-2)^2 + y^2 = 4$.

17. (12 points) Evaluate the integral $\displaystyle\int_{-3}^{0}\int_{0}^{\sqrt{9-y^2}}\int_{0}^{\sqrt{9-x^2-y^2}} (4z)\; dz\, dx\, dy$ in spherical coordinates.

18. (12 points) Find the work done by the field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$ over the path $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq \dfrac{\pi}{2}$.

19. (12 points) Use Green's Theorem to evaluate the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \left\langle 6y + \frac{\sin^2 x}{10+x^4}, 3y^9\cos(y^2) - 4x \right\rangle$ and C is the positively oriented triangle shown below.



20. (12 points) Find the surface area of the paraboloid $z = 5 - 2x^2 - 2y^2$ that lies above the plane $z = -13$.

21. (16 points). Let $\mathbf{F} = (xy^2 - z)\mathbf{i} + (12x + yz^2)\mathbf{j} + (zx^2 - \sin x)\mathbf{k}$ and let S be the sphere $x^2 + y^2 + z^2 = 4$. Use the Divergence Theorem to find evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.