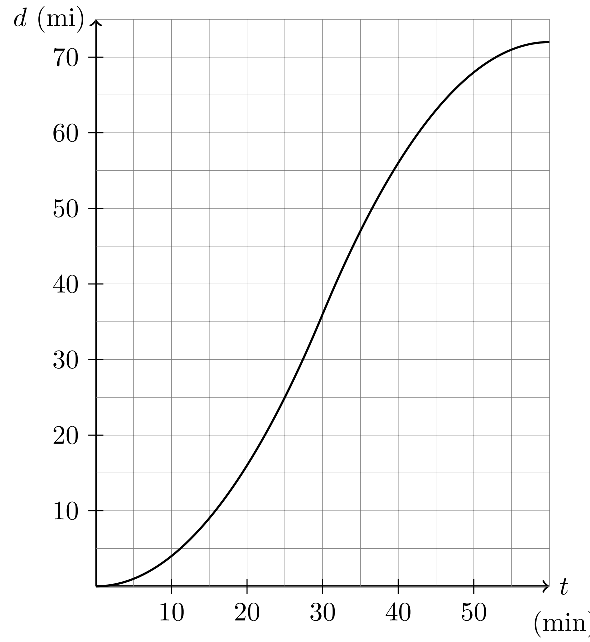
Chapter 2
Derivatives

Contents

Motivation to Chapter 2

At the beginning of chapter 1 we were obsessed with slope. Though after section 4 we kind of took a detour and talked more about limits and all the great things you can do with them. Well now it is officially time to start thinking about slope and how quantities change again. Though more than just an average rate of change, in chapter 2 we want to think about instantaneous rate of change. Again think back to this example.

Example: You are driving from Lansing to Detroit. To the right is a graph representing your distance from Lansing. At what time is your speed the greatest?



And although I said it already, it is worth repeating. Slope is worthy of our focus because it helps us determine how fast things are changing. In the above example, how fast the distance is changing (aka velocity). More generally the (instantaneous) rate of change is something worth studying, not just for velocity sake but so much more! Other examples include:

- Rate of increase/decrease of stock values.
- Infection rate of a disease.
- Rate of a chemical reaction.
- Rate of population growth.
- Rate of change of velocity (acceleration).
- Rate of change for monthly sales at your business.
- Rate at which a temperature changes.

1 Derivatives and Rates of Change

1.1 VIDEO - Definitions

Objective(s):

- Define the derivative of a function at a specific point and recall its graphical interpretation.
- Derive the equation of the tangent line.

Back in 1.4 we brought up the first big idea of calculus, tangent lines and slope at a point. We got kind of distracted in Chapter 1 playing with limits but now it is finally time to apply our limit knowledge to calculating slopes at a point.

Definition(s) 1.1. The ____________________ **of a function** f **at a number** c, denoted by _________, is the number

$$f'(c) =$$

if the limit exists. An equivalent formulation that is used more often is

$$f'(c) =$$

Definition(s) 1.2. The tangent line to the graph of $y = f(x)$ at the point $(c, f(c))$ is the line through ______________ whose slope is equal to _________.

Theorem 1.3. Point slope form shows that the equation of the tangent line for $y = f(x)$ through the point $(c, f(c))$ is given by

Example 1.4. Find $f'(3)$ for the function $f(x) = 2x + 5$

1.2 VIDEO - Examples and Applications of the Derivative

Objective(s):

- Get practice calculating the derivative.
- Go over some applications of the derivative.

Example 1.5. Calculate $f'(3)$ for $f(x) = \sqrt{x+1}$

Remark 1.6. Because limits do not always exist and the derivative is defined by a limit the derivative may not exist. At such a point the function is called ______________________________.

Example 1.7. Show that $f(x) = |x-1|$ is not differentiable when $x = 1$.

Remark 1.8. Recall from 1.4 that the average and instantaneous rate of change of $f(x)$ has the units:

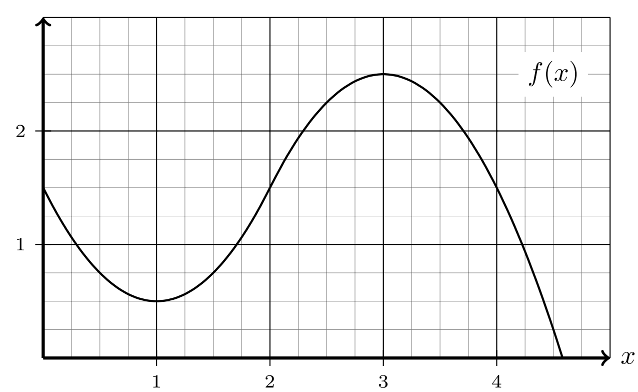
$$\frac{\text{units of } f}{\text{units of } x}$$

Therefore the derivative will have these same units.

Example 1.9. Suppose I throw a ball upward at an initial speed of 30 ft/s. From our favorite physics class we know that the height of the ball is given by $s = 4 + 30t - 16t^2$ feet at time t seconds after I let go. Find the ball's instantaneous velocity 1 seconds after I let go.

Example 1.10. Use the graph of $f(x)$ and grid shown to the right to approximate

(a) $f'(3)$



(b) $f'(4)$

2 The Derivative as a Function

2.1 VIDEO - Definitions and Calculations

Objective(s):

- Calculate the formula for the derivative given a function.

So far we have been finding derivatives at specific points but if you put all those points together you start to get an entirely new function called the derivative function. Let's see what I mean by checking out the graph:

https://www.desmos.com/calculator/becz6ylyq9

Definition(s) 2.1. The ____, denoted by ____, is the function

Another common notation (called Leibniz notation) is to write $\dfrac{df}{dx}$ or $\dfrac{d}{dx}f(x)$ instead of $f'(x)$.
The derivative at $x = c$ in this notation is written $\left.\dfrac{df}{dx}\right|_{x=c}$.

Example 2.2. Compute the derivative using the limit definition for $f(x) = x^2 + x - 8$.

Example 2.3. Compute the derivative using the limit definition for $g(x) = \dfrac{5}{x}$.

Definition(s) 2.4. A function f is ________________________________ if $f'(c)$ exists.

It is ________________________ on the _____________________ if it is differentiable at every number in (a, b).

2.2 VIDEO - Why Not Differentiable?

Objective(s):

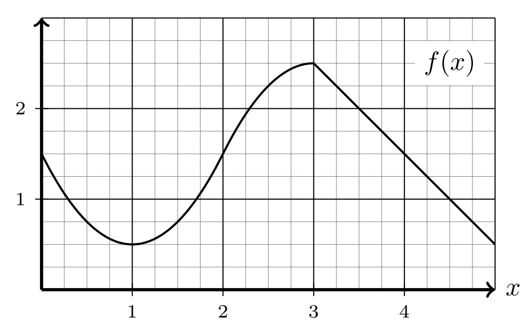
- Investigate ways in which a function can fail to be differentiable.

As we saw in **Example 1.7** there are ways in which a function can fail to be differentiable. In this video we will explore a variety of ways this can happen.

Case 1: Jagged Edges (Cusp)

Because the derivative is a two sided limit (as $h \to 0$) if the left and right hand limits do not agree the function will not be differentiable. We saw this in **Example 1.7**. Graphically these look like jagged edges. Let's see another example

Example 2.5. Below is a graph of $f(x)$. Determine any points where $f(x)$ is not differentiable and explain why it fails to be differentiable there.



Case 2: Not Continuous

In this case the left or right hand limit (as $h \to 0$) will not be real (finite) number. Graphically and algebraically we know how to spot discontinuities. This also leads to an important theorem:

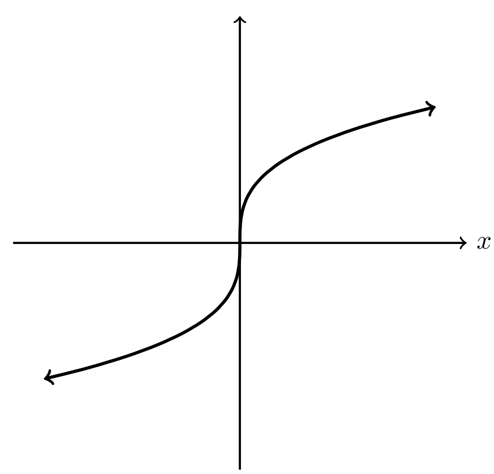
Theorem 2.6. If f is differentiable at c, then f is continuous at c.

Example 2.7. Consider the function $f(x) = \begin{cases} x+1 \text{ if } x < 3 \\ x+2 \text{ if } x \geq 3 \end{cases}$. Show that $f(x)$ is not differentiable at $x = 3$.

Case 3: Vertical Tangent

The final case we will consider is when the tangent line is vertical so the slope is not a real (finite) number.

Example 2.8. Consider the graph of $g(x) = \sqrt[3]{x}$ below. Sketch the tangent line at $x = 0$ to see that $g'(0)$ does not exist.



2.3 VIDEO - A Graphical Viewpoint

Objective(s):

- Given a graph of a function sketch the graph of its derivative.

As we saw at the beginning of this section computers are quite good at graphing the derivative function given the original function. We would like to gain this skill as well.

Remark 2.9 (Tips for sketching the derivative function $f'(x)$ given the graph of $f(x)$)**.**

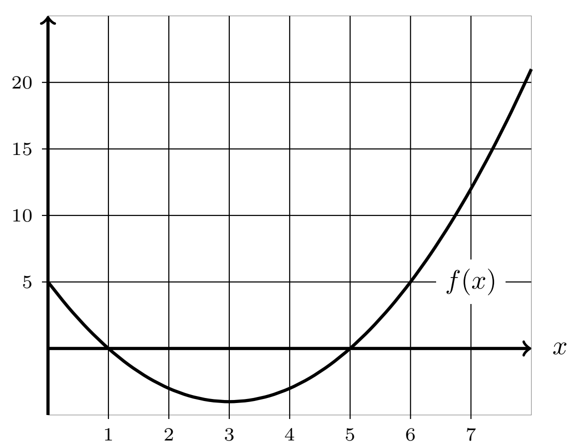
(a) Find where the derivative is ________.

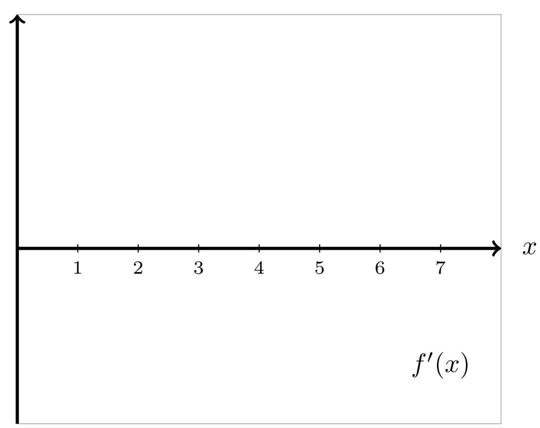
(b) Determine intervals where $f(x)$ is ____,
$f'(x)$ will be ________________________________ on these intervals.

(c) Estimate the instantaneous rate of change at ______________________ in each interval.

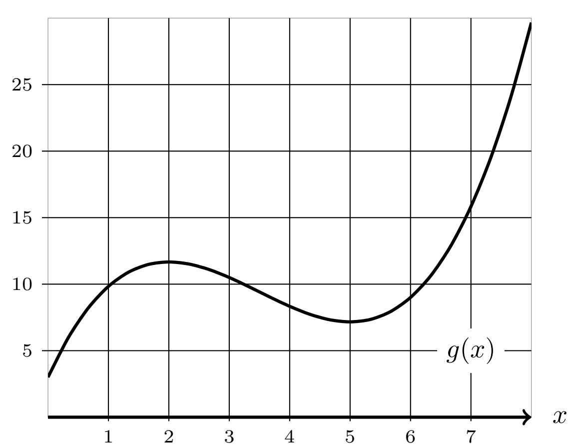
Example 2.10. Given the plot of each of the functions below sketch their derivatives.

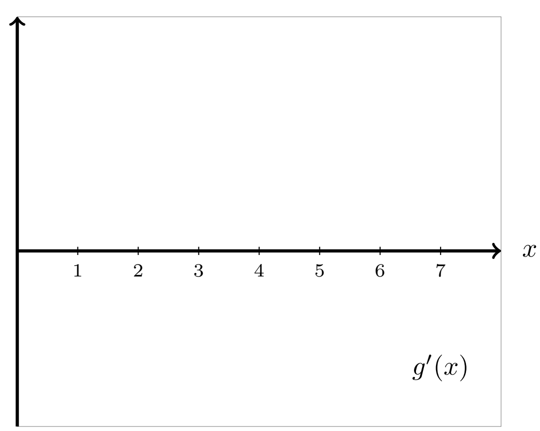
(a)





(b)





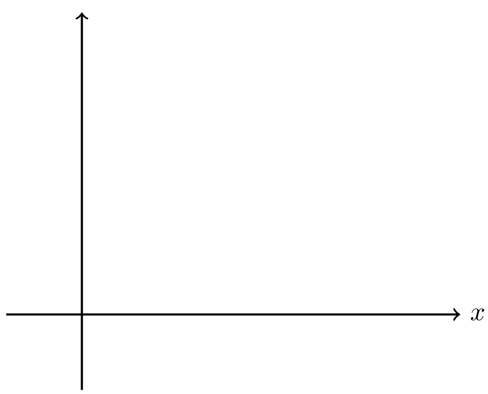
3 Differentiation Formulas

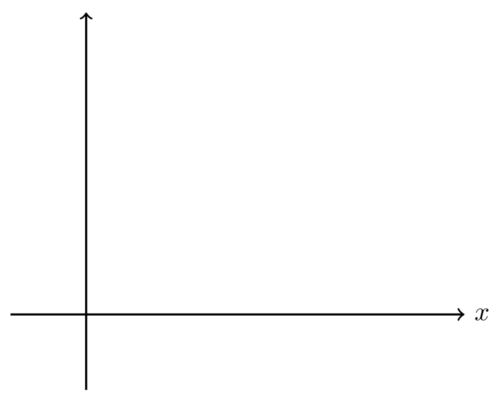
As we saw in 2.1 and 2.2 using the limit definition of the derivative takes some time and can be quite difficult. It is important that we don't forget this technique (as it is a major learning objective for the course) but now we will transition into learning the rules that make taking derivatives ***much*** easier.

3.1 VIDEO - Constants, Lines, and Powers

Objective(s):

- Understand the constant and linear differentiation rules and why they make sense.
- Understand the power rule and how powerful it is!
- Apply these rules to a variety of differentiation problems





Theorem 3.1. If $f(x) = c$ then $f'(x) =$

Theorem 3.2. If $f(x) = mx + b$ then $f'(x) =$

Example 3.3. Find the derivatives for the following functions.

(a) $f(x) = 7$

(b) $g(t) = 13 - 2t$

(c) $h(x) = \dfrac{4x^2 - 1}{2x + 1}$

Okay now we are moving into the power rule which is very powerful and we will use it all the time for the rest of the course but before I give it to you we need to review some algebra.

Definition(s) 3.4. If n is any real number then

$$\frac{1}{x^n} = \qquad\qquad \sqrt[n]{x} =$$

where defined

Theorem 3.5 (Power Rule)**.** If $f(x) = x^n$ for some real number n then $f'(x) =$

Example 3.6. Take the derivatives of the following functions

(a) $p(x) = x^3$

(b) $f(x) = x^{0.2}$

(c) $g(x) = x^{5/2}$

(d) $h(x) = \frac{1}{x}$

(e) $F(x) = \frac{1}{x^3}$

(f) $G(x) = \sqrt{x}$

(g) $H(x) = \sqrt[3]{x}$

But still you see these are pretty simple functions. What about when you start combining them with addition, subtraction, multiplication and division?

3.2 VIDEO - Sums, Differences, and Constant Multiples

Objective(s):

- Understand the statements of the sum, difference, and constant multiple rules.
- Apply these rules to a variety of differentiation problems

Luckily the rules here are exactly what you would expect.

Theorem 3.7 (Constant Multiple, Sum, and Difference Rules)**.** Take c to be a constant and f, g both differentiable functions, then:

$$\frac{d}{dx}\left(cf(x)\right) = \left(cf(x)\right)' =$$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \left(f(x)+g(x)\right)' =$$

$$\frac{d}{dx}\left(f(x)-g(x)\right) = \left(f(x)-g(x)\right)' =$$

Example 3.8. Calculate the derivatives of the following functions

(a) $f(x) = -7\sqrt{x}$

(b) $g(x) = x^2 + \sqrt[3]{x} + 2x$

(c) $h(x) = \dfrac{5}{x} - x^{1.7} + 4$

3.3 VIDEO - Products and Quotients

Objective(s):

- Understand the statements of the product and quotient rules.
- Apply these rules to a variety of differentiation problems

However the product and quotient rules are perhaps less expected.

Theorem 3.9 (Product, Quotient)**.** Take f, g both differentiable functions, then:

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left(f(x)g(x)\right)' =$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \left(\frac{f(x)}{g(x)}\right)' =$$

Example 3.10. Differentiate $\left(x^2 - \frac{3}{x}\right)(\sqrt{x} + x + 3)$.

Example 3.11. Find the equation of the tangent line to the curve $y = \frac{2x}{x+1}$ a the point $(1, 1)$.

4 Derivatives of Trigonometric Functions

4.1 VIDEO - Proofs and Examples of Some Trigonometric Derivatives

Objective(s):

- Prove some important trig derivatives
- Apply trig derivative to a few examples.

Theorem 4.1.

(a) $\dfrac{d}{dx}(\sin(x)) =$

(b) $\dfrac{d}{dx}(\cos(x)) =$

Proof of (a):

Example 4.2. Derive formulas for the derivatives of the following functions.

(a) **Put to memory:** $\dfrac{d}{dx}\left(\tan(x)\right)$

(b) **Put to memory:** $\dfrac{d}{dx}\left(\sec(x)\right)$

(c) **Extra**: $\dfrac{d}{dx}\left(\csc(x)\right)$

(d) **Extra**: $\dfrac{d}{dx}\left(\cot(x)\right)$

4.2 VIDEO - Proofs and Examples of Some Trigonometric Limits

Objective(s):

- Prove some important trig limits
- Apply these trig limits to a few examples.

Theorem 4.3. $\lim_{x \to 0} \frac{\sin x}{x} =$ $\lim_{x \to 0} \frac{\cos x - 1}{x} =$

Idea of Proof that $\lim_{x \to 0} \frac{\sin x}{x} =$

More generally,

Theorem 4.4. If $\lim_{x\to a} f(x) = 0$ then

$$\lim_{x\to a} \frac{\sin(f(x))}{f(x)} =$$

And likewise

$$\lim_{x\to a} \frac{f(x)}{\sin(f(x))} =$$

From these we get many new types of limit problems. Lets try a few before moving on to derivatives.

Example 4.5. Find the limit $\lim_{x\to 0} \dfrac{\sin(5x)}{3x}$

Example 4.6. Find the limit $\lim_{t\to 0} \dfrac{\tan(2t^2)}{4t}$

5 The Chain Rule

5.1 VIDEO - Review and the Main Theorem

Objective(s):

- Identify when a function can be decomposed into a composition of two functions.
- Understand the statement of the chain rule and how to apply it.

So far we know how to take the derivative of the sum, difference, product, and quotient of two functions but there was another thing functions sometimes do... eat eachother.

Example 5.1. Consider $f(x) = x^2$ and $g(x) = 3x + 1$. Find $f(g(x))$.

Definition(s) 5.2. In the above example we calculated $f(g(x))$ which is called the ____________________ of f and g. It is also sometimes denoted as $(f \circ g)(x)$.

So how do we take the derivative of these things? Well the first step is to be able to go backwards, a function that is really the ____________________ of two functions we need to be able to express it as such.

Example 5.3. Consider the function $h(x) = \sin(x^2 + x)$. Find functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

Now we are ready to take the derivative!

Theorem 5.4 (Chain Rule)**.** If g is differentiable at x and f is differentiable at $g(x)$, then the composition $F =$ ________ defined by __________________________ is differentiable at x and F' is given by

In Leibniz notation if we let $u = g(x)$, the same thing can be written as

Remark 5.5. Although the Leibniz notation should not literally be thought of as a fraction, it still gives a good way to remember the chain rule: it looks like the du's just cancel.

Example 5.6. Find the derivative of $h(x) = \sin(x^2 + x)$ from **Example 5.3**.

(a) Using the standard notation.

(b) Using the Leibniz notation.

5.2 VIDEO - Practicing the Chain Rule

Objective(s):

- Become more comfortable applying the chain rule to a variety of problems.

Example 5.7. Calculate the derivatives of the following functions by first writing them as a composition of simpler functions, and then applying the chain rule:

(a) $f(x) = \left(7x^3 + 2x^2 - x + 3\right)^5$

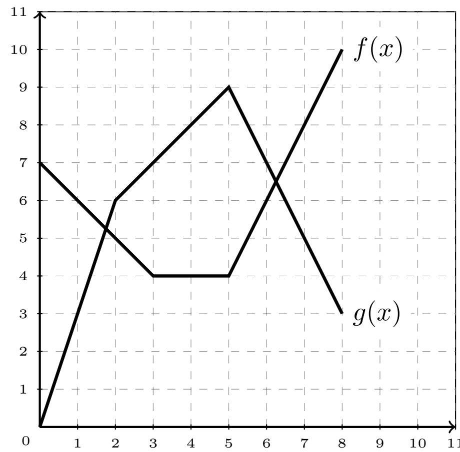
(b) $g(x) = \tan\left(3x^2 + 5x + 8\right)$

Example 5.8. A table of values for f, g, f', and g' is given:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(1)$.

(b) If $H(x) = g(f(x))$, find $H'(1)$.



Example 5.9. The graphs of two functions, f and g, are pictured below.

Let $h(x) = f(g(x))$, and $p(x) = g(f(x))$.

Compute the following derivatives, or explain why they do not exist:

(a) $p'(1)$

(b) $h'(2)$

Example 5.10. Quickly calculate the derivatives of the following functions:

(a) $y = \sqrt{x + \sqrt{x}}$

(b) $F(x) = \left(4x - x^2\right)^{100}$

7 Rates of Change in the Natural and Social Sciences

This section has word problems. I think one of the most difficult things about word problems is figuring out what to do!? So we will emphasize the mathematical equations and inequalities behind common phrases to help us solve these real world examples.

7.1 VIDEO - Moving Forward, Backward, and Not at All

Objective(s):

- Use differentiation to solve when an object or particle is moving forward, backward, or standing still.
- Find out when an object reaches its maximum height (in vertical motion problems).

First some review:

Definition(s) 7.1. The ____ of $y = f(x)$ **with respect to** x is the _______ of the tangent line (a.k.a. derivative). Using Leibniz notation, we write:

$$\frac{dy}{dx} =$$

As was brought up in Section 2.1, the units for dy/dx are the ______________ divided by the ______________.

Remark 7.2 (From Section 2.2)**.** If $s = f(t)$ is the position function of an object that is moving in a straight line, then $v(t) = s'(t)$ represents the ____________ at time t. Also, $a(t) = v'(t) = s''(t)$ is the ___________________ of the object at time t.

Definition(s) 7.3. Suppose that $s(t)$ is the position of an object at time t and $v(t)$ is its velocity at time t then:

(a) The object moving forward means ___________.

(b) The object moving backward means ___________.

(c) The object standing still means ___________.

(d) If the object is thrown upward and is acted upon by gravity then the object reaches its maximum height when ___________.

Let's try to visualize (d) before attempting a problem or two: `https://www.desmos.com/calculator/hkzgcmvj1b`

Example 7.4. A particle moves according to the law of motion $s(t) = \cos(\pi t/4)$ with $0 \leq t \leq 10$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.

(b) When is the particle at rest?

(c) When is the particle moving in the positive direction?

Example 7.5. The height (in meters) of a projectile shot vertically upward from a point 15 m above ground level with an initial velocity of 10 m/s is $h = 15 + 10t - 5t^2$ after t seconds.

(a) When does the projectile reach its maximum height?

(b) What is the maximum height?

(c) When does it hit the ground?

(d) With what velocity does it hit the ground?

7.2 VIDEO - Speeding Up and Slowing Down

Objective(s):

- Recall the definition of speed.
- Use differentiation to determine when an object is speeding up or slowing down.

Definition(s) 7.6. Speed is the ________________ of velocity. That is, the speed of an object at time t is given by ________.

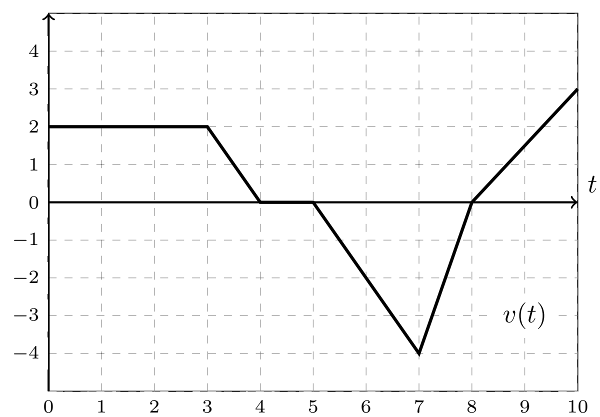
Definition(s) 7.7. For an object

(a) Speeding up means the ____________________________. That is __________________.

(b) Slowing down means the ____________________________. That is __________________.

These are okay definitions. Let's see how we can use them.

Example 7.8. The figure below shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in a closed interval $[0, 10]$.



(a) When is the particle speeding up?

(b) When is the particle slowing down?

But sometimes these definitions are horrible to use. Consider the following problem.

Example 7.9. When $t > 0$ a particle is moving along a straight line with velocity given by $v(t) = \dfrac{1-t^2}{(1+t^2)^2}$. When is the particle speeding up?

And its problems like these that make us think we are on an infomercial saying "*There's got to be a better way!*"

Theorem 7.10. Suppose at time t an objects velocity is given by $v(t)$ and its acceleration is given by $a(t)$ then:

(a) If ________________ then it is neither speeding up nor slowing down at t_1.

(b) If $a(t_1)$ and $v(t_1)$ have ________________________ then the object is speeding up at t_1.

(c) If $a(t_1)$ and $v(t_1)$ don't have ________________________ then the object is slowing down at t_1.

Example 7.11. When $t > 0$ a particle is moving along a straight line with position given by $s(t) = \dfrac{t^3}{3} - 3t^2 + 5t$. When is the particle slowing down?

And finally let's solve **Example 7.9**!

Example 7.7. When $t > 0$ a particle is moving along a straight line with velocity given by $v(t) = \dfrac{1-t^2}{(1+t^2)^2}$ and acceleration given by $a(t) = \dfrac{2t(t^2-3)}{(1+t^2)^3}$. When is the particle speeding up?

6 Implicit Differentiation

6.1 VIDEO - Intros and Ideas

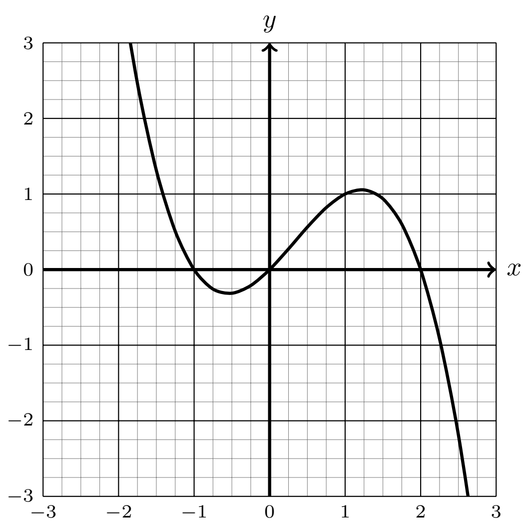
Objective(s):

- Introduce implicit differentiation.
- Recognize the type of problems that implicit differentiation solves.

So far everything we have differentiated has been a function. However there are plenty of curves that are not functions. Let's take a look at some side by side examples.

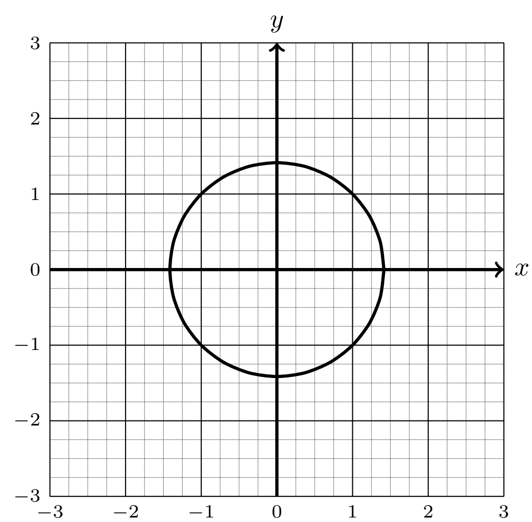
Example 6.1.

Use the graph of $y = -0.5x^3 + 0.5x^2 + x$ to approximate the slope of the tangent line at $(1, 1)$.



Now use calculus to find the slope of the tangent line to $y = -0.5x^3 + 0.5x^2 + x$ at $(1, 1)$.

Use the graph of $x^2 + y^2 = 2$ to approximate the slope of the tangent line at $(1, 1)$.



Now use calculus to find the slope of the tangent line to $x^2 + y^2 = 2$ at $(1, 1)$.

Definition(s) 6.2. Implicit Differentiation is a method of differentiating both sides of an equation with respect to x and then solving the resulting equation for y'.

6.2 VIDEO - Examples

Objective(s):

- Find the slopes of various curves by applying implicit differentiation.

Remark 6.3. Be careful that you are applying power, product, quotient, and chain rules correctly.

Example 6.4. Consider the curve $x^3 + y^3 = 6xy$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find an equation of the tangent line at the point $(3, 3)$.

(c) At what point in the first quadrant is the tangent line horizontal?

Example 6.5. Use implicit differentiation to find an equation of the tangent line to the curve $y \sin 2x = x \cos 2y$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

8 Related Rates

8.1 VIDEO - Round 1 – Fight!

Objective(s):

- Translate sentences into mathematical equations.
- Apply implicit differentiation and the chain rule to solve many types of related rates problems.

Remark 8.1 (General Technique for solving Related Rate Problems)**.**

(a) Relate the given quantities with an ________________.

(b) ________________________ the equation (using implicit differentiation and the chain rule).

(c) Solve for some __________________________ (derivative).

Remark 8.2. Geometric/Trig formulas you are expected to know

- **Circles**

$$C = \pi d \qquad d = 2r \qquad A = \pi r^2$$

- **Triangles**

$$A = \frac{1}{2}bh \qquad c^2 = a^2 + b^2 \text{ (for right } \Delta\text{s)}$$

$$\sin\theta = \frac{O}{H} \text{ (for right } \Delta\text{s)} \qquad \cos\theta = \frac{A}{H} \text{ (for right } \Delta\text{s)} \qquad \tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta} \text{ (for right } \Delta\text{s)}$$

- **Rectangles (Squares)**

$$P = 2l + 2w \qquad A = lw$$

- **Spheres**

$$V = \frac{4}{3}\pi r^3 \qquad SA = 4\pi r^2$$

- **Cones**

$$V = \frac{h}{3}(\text{area of base})$$

- **Cylinders**

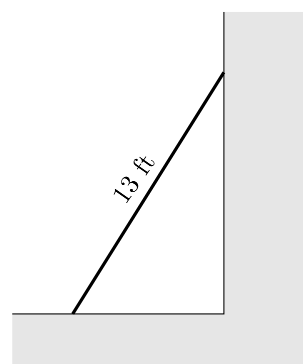
$$V = h(\text{area of base})$$

Example 8.3. A sphere is growing, its volume increasing at a constant rate of 10 in^3 per second. Let $r(t)$, $V(t)$, and $S(t)$, be the radius, volume, and surface area of the sphere at time t. If $r(1) = 2$, then compute:

(a) $r'(1)$

(b) $S'(1)$

Example 8.4. A 13 foot ladder is leaning against a wall, and is sliding down, with the top of the ladder moving downwards along the wall at 3 ft/sec. How fast is the bottom of the ladder moving away from the wall when the bottom is 5 ft from the base of the wall?

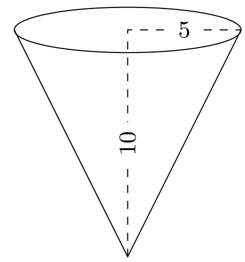


8.2 VIDEO - Round 2 – Fight!

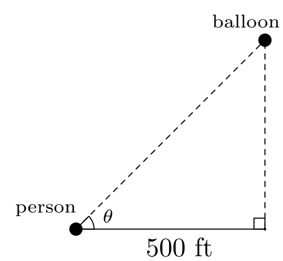
Objective(s):

- Translate sentences into mathematical equations.
- Apply implicit differentiation and the chain rule to solve many types of related rates problems.

Example 8.5. A cone-shaped tank is filling with water at a constant rate of 9 ft^3/min. The tank is 10 ft tall, and has a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?



Example 8.6. A balloon is rising vertically above a field, and a person 500 feet away from the spot on the ground underneath the balloon is watching it, measuring the angle of inclination, θ. When the angle is $\pi/4$ radians, the angle is increasing at $\frac{1}{10}$ radians per minute. At that moment, how fast is the balloon rising?



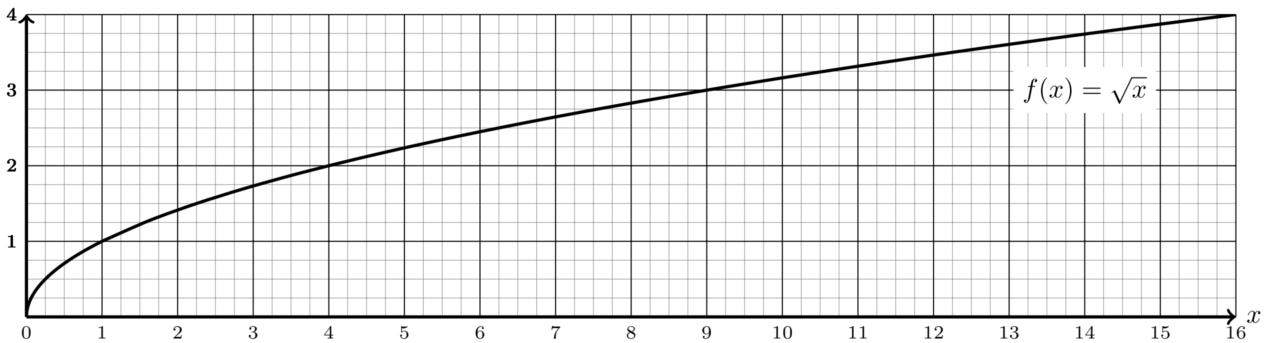
9 Linear Approximations and Differentials

9.1 VIDEO - Concepts and Definitions

Objective(s):

- Define the two new concepts of linearization and differentials.
- See how these concepts arise out of natural questions.

Consider the problem of trying to approximate $\sqrt{4.5}$. We know it is near $\sqrt{4} = 2$. So $\sqrt{4.5}$ should be like 2.1? 2.2 maybe? But these are just hunches... if only there was a way to use calculus to approximate values like this with more accuracy...



Example 9.1. Use the tangent line for $f(x) = \sqrt{x}$ at $x = 4$ to approximate $\sqrt{4.5}$.

Remark 9.2. ____________ approximation or ______________________ approximation are two names for using the equation of a tangent line to approximate a function

Definition(s) 9.3.

is called the _______________________ of f at a.

Remark 9.4. In general ____________________ so long as x is ________ to a.

There is a sort of "dual" topic to linearization which is ____________________. Which answer the questions. If a graph $y = f(x)$ is originally at $x = a$ how much does the y value change if your x value moves a little? (sometimes called Δx or dx)

Example 9.5. Use differentials to approximate how much does the value of $y = \sqrt{x}$ change as x moves from 4 to 4.5?

before we answer let's take a look at the concept at `https://www.desmos.com/calculator/v0xyiudz8o`

Remark 9.6. An equivalent notion to linearization is differentials. Consider the definitions:

Where dy and dx are considered variables in their own right.

Definition(s) 9.7.

$$dx = \Delta x = x - a \qquad\qquad \Delta y = f(x + \Delta x) - f(x)$$

Remark 9.8.

$$\approx$$

9.2 VIDEO - A Few More Examples

Objective(s):

- Utilize the tangent line or differentials to estimate how a function is changing around a specific point.

Example 9.9. Consider the function $f(x) = \sin x$.

(a) Find the linearization $L(x)$ of the function at $a = \pi/4$.

(b) Use the linearization to approximate $\sin(11\pi/40)$

Example 9.10. Use linear approximation to estimate $\sqrt[3]{1001}$

Example 9.11. Consider the curve $y = \cos(\pi x)$.

(a) Find the differential dy of

(b) Evaluate dy for $x = 1/3$ and $dx = -0.02$

Example 9.12. Use a differential to approximate how much the value of $y = \sqrt{x^2 + 8}$ changes as x moves from 1 to 1.02.