MTH 133 Exam 2 Information Sheet

Taylor and Maclaurin Series

You are responsible for knowing / memorizing at least the following four (4) Maclaurin series expressions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

*i.e. the **even**-powered terms*

$$sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

*i.e. the **odd**-powered terms*

In general, any differentiable function *f(x)* can have a power series (or Taylor series) representation as:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Where $f^{(n)}(a)$ represents the n^{th} derivative of *f* evaluated at the center *a*.

Convergence Tests

When determining the convergence or divergence of a series, you need to show the following three pieces of information:

1. State the test you are using
2. Show how all the hypothesis / assumptions of the tests are fulfilled
3. State the conclusion of the test

Example 1a

Determine if the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Solution

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ *is a* $p-$*series with* $p = 2 > 1$, *therefore converges*. $\blacksquare$

Example 1b

Determine if the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{3n^2}{2n^4+n}$$

Solution

$$\sum_{n=1}^{\infty} \frac{3n^2}{2n^4+n} < \sum_{n=1}^{\infty} \frac{3n^2}{2n^4} = \frac{3}{2}\sum_{n=1}^{\infty} \frac{1}{n^2}$$

Larger denominator → smaller fraction

which $\sum_{n=1}^{\infty} \frac{1}{n^2}$ *is a* $p-$*series with* $p = 2 > 1$, *therefore converges*

Since the original series is less than a converging series, this original series also converges by the direct comparison test. $\blacksquare$

Note that Example 1b essentially has two problems: the p-series problem, Example 1a, and then the direct comparison problem, Example 1b. For full credit, EVERY TIME you use a test, you will need to display the three pieces of information. The direct comparison test has this hypothesis to satisfy:

> You must show that the original series is smaller than a known converging series or larger than a known diverging series.

In order to show the convergence of the "known" series, you will have to use another convergence test and also fulfill the hypotheses of that test. The p-series test was used just to satisfy the hypotheses of the direct comparison test.

Lastly, as a small side note, in the last step of Example 1b, the series:

$$\frac{3}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$$

is not just a plain p-series. However, any constant multiple (other than 0) of a series does not affect the convergence or divergence of the series so you can use the test as shown above. This minor point isn't necessary for full credit on the test.