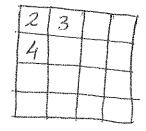
The 29th Herzog Competition

Problem 1. Let p and q be two consecutive odd primes (like 5 and 7 or 23 and 29). Prove that p + q can be represented as a product of 3 integer factors, each of the factors being greater than 1.

Problem 2. An n by n square filled with numbers $1, 2, ..., n^2$ is called magic if the sum of the numbers in each row, each column, and each of the two diagonals is the same. Does there exists a magic 4 by 4 square that contains numbers 2, 3, and 4 in the positions shown below?



Problem 3. During the last 11 weeks John played at least one game of chess every day but not more than 12 games during any week. Prove that there was a period of several consecutive days during which John played exactly 22 games.

Problem 4. Find (with proof) all mappings F from \mathbb{Z}_{239} (the ring of residues modulo 239) into itself satisfying

$$F(x+y) = F(x)F(y)$$
 for all $x, y \in \mathbb{Z}_{239}$.

Problem 5. Prove that for every x > 0,

$$e^{-x} + e^{-1/x} \ge \frac{2}{e}.$$

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